

Introduction to Impossible Probability, Statistics of Probability or Probabilistic Statistics. VOL 7

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Introduction to Impossible Probability,
Statistics of Probability or Probabilistic Statistics
Dr Rubén García Pedraza
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Translated into English in September 2025,
using ChatGPT, keeping the translation as faithful as possible to the original

Preliminary Words [2015 edition]

Introduction to Impossible Probability, Statistics of Probability or Probabilistic Statistics is one of those works in which a deep bond between mathematics and philosophy can be found.

It always asks about the ultimate purpose of science, wrapped in a veil of uncertainty and relativity.

There is always a halo of indeterminism, since in the end it is chance itself—the random variations of what we call reality, although we do not quite know what it is—that becomes the cause of causes.

From an interpretation not devoid of a certain nihilism, it offers a different vision of mathematics applied to statistics and probability.

The work is organized into a series of sections whose exposition unfolds in a continuous manner, each section and each paragraph being numbered in an ordered numerical sequence, in a way that facilitates reading.

Introduction to Impossible Probability first appeared in December 2011, in a limited edition of which a few copies still remain. It was later modified into a new eBook version that appeared in January 2013, whose first revision dates from January 2014. The version now presented is the latest update, based on contributions from January 2015.

To all readers, many thanks, and enjoy,
Rubén García Pedraza

Preliminary Words to the English Translation

The work presented here, *Introduction to Impossible Probability*, is essential for understanding the original meaning of the Global Artificial Intelligence. Its origins can be traced back to my writings in 2002 on Astro-ecology, the science that understands the cosmos as a single environment that must be studied from a unified, single science. In essence, this is what the Global Artificial Intelligence seeks to achieve. At the same time, in 2002 I developed the formulations of Impossible Probability, beginning in the early hours of September 11, 2001 with the *Impact of the Defect*, which was later expanded into the *Second Method* on October 16, 2002.

In essence, the entire original philosophy is based on the Cartesian idea of the necessity of a margin of doubt in any scientific proposition. This Cartesian margin of doubt would later, in the application of Karl Popper's falsificationism to inferential statistics, be understood as *critical reason*, the necessary and acceptable margin of error in any provisionally true proposition.

Impossible Probability maintains and deepens the Cartesian and Popperian rationalist tradition, enriching it with new philosophical contributions and some mathematical formulations. It rests on the understanding that the necessity of a margin of doubt in the existence of every provisional truth stems from the human limitation of being incapable of accessing the noumenon of things. It is this a priori limitation—the metaphysical limitation of the human condition—that requires humans to accept errors. Yet this does not mean that the noumenon cannot be known by other means. For this reason, in *Impossible Probability* we propose the construction of the Global Artificial Intelligence: a machine capable of making real the dream of a unified science, a single science that may lead us to the reason for all things. The creation of a global matrix, which may ultimately be resolved in a single equation, will provide the equation that solves the matrix, the very reason for everything.

Due to the special conditions in which I find myself—explained in other documents available on Internet Archive and Amazon—I am compelled to publish as quickly as I translate. For this reason, the English version of *Introduction to Impossible Probability* will be released in volumes as the translation progresses.

September 1, 2025, London, Leytonstone, 18:32
Imposiblenever@gmail.com

“What did you do with the other guys

Did people brush their teeth

In the concentration camp”

Ruben Garcia Pedraza

Poem “*Mr Saddle*”

At Stratford Library Poetry Club

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14. Relative Descriptive Statistics

14.1. Foundations

14.1.1. If the logic of the Bias Level is the relationship of factors in order to know the bias of one factor over another—for example, empirical probability minus central tendency—and the central tendency may be described by alternative values, intermediate empirical probability, and empirical probability closest to the inversion of N .

14.1.2. A type of relative Bias Level could be the difference between any empirical value (empirical probability) and either the intermediate probability or the empirical probability closest to the inversion of N .

14.1.3. And if it is possible to set up a differential relationship between any empirical value and the intermediate empirical probability, or the one closest to the inversion of N , in order to know, for example, the dispersion of maximum or minimum values, then any empirical probability may be differentially related to the maximum or minimum empirical probability. One may then study the mean dispersion of the maximum or minimum relative to the other empirical probabilities, logically excepting themselves. That is, the average of the sum of all differentials of each empirical probability minus the maximum or the minimum, excepting from the summation the differential of the maximum with itself or of the minimum with itself. This implies an average of the sum of the differentials of each empirical probability minus the maximum, or the minimum minus each empirical probability, which will be an average insofar as the sum is divided by N minus one.

14.1.4. And if one can establish a differential relationship of the maximum or the minimum in relation to any other possible empirical probability, in order to know the degree of dispersion of any other probability with respect to the maximum or minimum, then one can also know the degree of possible dispersion of any empirical probability in relation to any other specific empirical probability $p(xn)$, except with itself. Thus, it would be the average of the sum of absolute differentials of the rest of the empirical probabilities minus that specific empirical probability, excepting the differential of that empirical probability with itself. The summation will then be averaged over N minus one, since of the N possible differentials one is excluded: the differential of the empirical probability with itself.

14.1.5. And to the same extent that one can establish a relative Bias Level of any empirical probability with respect to the whole sample (excluding itself), one can also study the sufficient tendency of any probability toward the maximum, the minimum, or the central tendency, whether in the study of bias or equality of opportunities.

14.1.6. Just as one can calculate the relative Bias Level, one could equally calculate the relative Similarity Level, although it is only mentioned here, since it will not be presented in detail. In *Introduction to Impossible Probability* priority is given to bias studies as they are the most easily adaptable to traditional statistics.

14.1.7. One of the main qualities of relative descriptive statistics is that natural goodness does not hold, which naturally only holds in normal descriptive statistics, understood as those calculated solely on the inversion of N . Therefore, in certain relative statistics—more specifically, descriptive statistics relative to the maximum or the minimum—it would not make much sense to calculate Variance or Standard Deviation, for the simple reason that there will be no negative differentials in any case, as long as the factors are ordered appropriately. In those relative statistics where negative differentials may indeed exist—for example, the relative Bias Level with respect to the intermediate or the closest to inversion—then, for the calculation of their relative dispersions, one may choose either the sum of the absolute values of the differentials, or the sum of the squared values of the differentials, or the square root of the average of the squared differentials.

14.1.8. In the following models of rational criticism that are presented below, in order not to extend the exposition too much, only models of rational criticism through critical proportions are shown, as these most facilitate rational criticism.

14.2. Descriptive Statistics of the Maximum “ $p(x_i+)$ ”

$p(x_i+) =$ maximum empirical probability

14.2.1. Relative Bias Level with respect to the maximum. Differential of the maximum minus each empirical probability.

Relative Bias Level with respect to the maximum =

$$p(x_i+) - p(x_i)$$

14.2.2. Mean Deviation relative to the maximum, $DM(+)$. Average of all differentials of the maximum minus each empirical probability, except for the differential of the maximum with itself. For this reason, the average is taken over N minus one, since in the whole sample there is only one differential that is not included in the summation (the differential of the maximum with itself).

Mean Deviation relative to the maximum =

$$DM(+) = \sum (p(x_i+) - p(x_i)) : (N - 1)$$

14.2.3. Variance relative to the maximum., $s^2(+)$ All differentials squared divided by N minus one, although it would be unnecessary because no differential will be negative.

Variance relative to the maximum =

$$s^2(+) = \sum (p(x_i+) - p(x_i))^2 : (N - 1)$$

14.2.4. Standard Deviation relative to the maximum. Square root of the Variance.

Standard Deviation relative to the maximum =

$$S(+) = \sqrt{[\sum (p(x_i+) - p(x_i))^2 : (N - 1)]}$$

14.2.5. Critical Proportion relative to the maximum.

14.2.5.1. Relative Proportion to the maximum, quotient of empirical probability over the maximum.

Relative Proportion to the maximum =

$$p(x_i) : p(x_i+)$$

14.2.5.2. Critical Proportion relative to the maximum.

14.2.5.2.1. Critical Proportion of equality relative to the maximum, quotient of empirical probability over the maximum, and the result minus critical probability. Zero or positive indicates sufficient tendency toward the maximum. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of equality relative to the maximum =

$[p(x_i) : p(x_{i+})] - p(x_c) = \text{zero or positive} \rightarrow \text{sufficient equality to the maximum}$

$$p(x_c) = X : 100$$

X = reliability percentage

14.2.5.2.2. Critical Proportion of bias relative to the maximum. Critical probability minus quotient of empirical probability over the maximum. Zero or positive indicates sufficient bias relative to the maximum. Critical probability equals error percentage X divided by one hundred.

Critical Proportion of bias relative to the maximum =

$p(x_c) - [p(x_i) : p(x_{i+})] = \text{zero or positive} \rightarrow \text{sufficient bias relative to the maximum}$

$$p(x_c) = X : 100$$

X = error percentage

14.3. Descriptive Statistics of the Minimum “ $p(x_{i-})$ ”

$p(x_{i-})$ = minimum empirical probability

14.3.1. Relative Bias Level with respect to the minimum. Differential of each empirical probability minus the minimum.

Relative Bias Level with respect to the minimum =

$$p(x_i) - p(x_{i-})$$

14.3.2. Mean Deviation relative to the minimum, $DM(-)$. Average of all differentials of each empirical probability minus the minimum, except for the differential of the minimum with itself. For this reason, the average is taken over N minus one, since in the whole sample there is only one differential that is not included in the summation (the differential of the minimum with itself).

Mean Deviation relative to the minimum =

$$DM(-) = \sum (p(x_i) - p(x_{i-})) : (N - 1)$$

14.3.3. Variance relative to the minimum, $s^2(-)$. All differentials squared divided by N minus one, although it would be unnecessary since there will be no negative differentials.

Variance relative to the minimum =

$$s^2(-) = \sum (p(x_i) - p(x_{i-}))^2 : (N - 1)$$

14.3.4. Standard Deviation relative to the minimum. Square root of the Variance.

Standard Deviation relative to the minimum =

$$S(-) = \sqrt{[\sum (p(x_i) - p(x_{i-}))^2 : (N - 1)]}$$

14.3.5. Critical Proportion relative to the minimum

14.3.5.1. Relative Proportion to the minimum, the minimum divided by the empirical probability.

Relative Proportion to the minimum = $p(x_{i-}) : p(x_i)$

14.3.5.2. Critical Proportion relative to the minimum.

14.3.5.2.1. Critical Proportion of equality relative to the minimum. Quotient of the minimum over empirical probability, and the result minus the critical probability. Zero or positive → accepted as tendency of the empirical probability toward the minimum. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of equality relative to the minimum =

$[p(x_i-) : p(x_i)] - p(x_c) = \text{zero or positive} \rightarrow \text{equality to the minimum accepted}$

$$p(x_c) = X : 100$$

X = reliability percentage

14.3.5.2.2. Critical Proportion of bias relative to the minimum. Critical probability minus quotient of the minimum over empirical probability. Zero or positive → accepted as relative bias to the minimum. Critical probability equals error percentage X divided by one hundred.

Critical Proportion of bias relative to the minimum =

$p(x_c) - [p(x_i-) : p(x_i)] = \text{zero or positive} \rightarrow \text{relative bias to the minimum accepted}$

$$p(x_c) = X : 100$$

X = error percentage

14.4. Descriptive Statistics of the Intermediate Probability “ $p(x_{i+/-})$ ”

14.4.1. The Intermediate “ $p(x_{i+/-})$ ”

$$p(x_{i+/-}) = (p(x_{i+}) + p(x_{i-})) : 2$$

14.4.2. Relative Bias Level with respect to the intermediate. Differential of each empirical probability minus the intermediate.

$$\text{Relative Bias Level with respect to the intermediate} = p(x_i) - p(x_{i+/-})$$

14.4.3. Mean Deviation relative to the intermediate, $DM(+/-)$. Average of the absolute value of all differentials of each probability minus the intermediate.

Mean Deviation relative to the intermediate =

$$DM(+/-) = \sum | p(x_i) - p(x_{i+/-}) | : N$$

14.4.4. Variance relative to the intermediate. Average of all squared differentials. Logically, in this case, the calculation of relative variance makes sense, since relative biases may be negative, as in the following relative statistics of subsequent individual statistics.

Variance relative to the intermediate =

$$s^2(+/-) = \sum (p(x_i) - p(x_{i+/-}))^2 : N$$

14.4.5. Standard Deviation relative to the intermediate. Square root of the Variance.

Standard Deviation relative to the intermediate =

$$S(+/-) = \sqrt{[\sum (p(x_i) - p(x_{i+/-}))^2 : N]}$$

14.4.6. Critical Proportion relative to the intermediate

14.4.6.1. Relative Proportion to the intermediate

14.4.6.1.1. Proportion of equality relative to the intermediate for the study of equality of opportunities. Absolute value of the relative Bias Level to the intermediate over the difference between one and the intermediate.

Proportion of equality relative to the intermediate =

$$|p(x_i) - p(x_{i+/-})| : (1 - p(x_{i+/-}))$$

14.4.6.1.2. Proportion of positive bias relative to the intermediate. Relative Bias Level to the intermediate over the difference between one and the intermediate.

Proportion of positive bias relative to the intermediate =

$$(p(x_i) - p(x_{i+/-})) : (1 - p(x_{i+/-}))$$

14.4.6.1.3. Proportion of negative bias relative to the intermediate. Intermediate minus empirical probability, and the result over the intermediate.

Proportion of negative bias relative to the intermediate =

$$(p(x_{i+/-}) - p(x_i)) : p(x_{i+/-})$$

14.4.6.2. Critical Proportion relative to the intermediate

14.4.6.2.1. Critical Proportion of equality relative to the intermediate. Critical probability minus the quotient of the absolute value of the relative Bias Level to the intermediate over the difference between one and the intermediate. Zero or positive → equality accepted. Critical probability equals error percentage X divided by one hundred.

Critical Proportion of equality relative to the intermediate =

$$p(x_c) - [| p(x_i) - p(x_{i+/-}) | : (1 - p(x_{i+/-}))] = \text{zero or positive} \rightarrow \text{equality relative to the intermediate} \quad \text{accepted}$$

$$p(x_c) = X : 100$$

X = error percentage

14.4.6.2.2. Critical Proportion of positive bias relative to the intermediate. Relative Bias Level to the intermediate over the difference between one and the intermediate, and the result minus the critical probability. Zero or positive → sufficient positive bias relative to the intermediate accepted. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of positive bias relative to the intermediate =

$$\left[\left(p(x_i) - p(x_{i+/-}) \right) : \left(1 - p(x_{i+/-}) \right) \right] - p(x_c) = \text{zero or positive} \rightarrow \text{positive bias relative to the intermediate} \quad \text{accepted}$$

$$p(x_c) = X : 100$$

X = reliability percentage

14.4.6.2.3. Critical Proportion of negative bias relative to the intermediate. Difference of intermediate minus empirical probability, result over the intermediate, minus critical probability. If equal to or greater than the critical probability → accepted, thus zero or positive → accepted. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of negative bias relative to the intermediate =

$$\left[\left(p(x_{i+/-}) - p(x_i) \right) : p(x_{i+/-}) \right] - p(x_c) = \text{zero or positive} \rightarrow \text{negative bias relative to the intermediate} \quad \text{accepted}$$

$$p(x_c) = X : 100$$

X = reliability percentage

14.5. Descriptive Statistics of the Probability Closest to the Inversion of N

$p(x_{i\approx})$ = empirical probability closest to the inversion of N

14.5.1. Relative Bias Level with respect to the closest to inversion of N. Differential of each empirical probability minus the closest to inversion of N.

Relative Bias Level with respect to the closest = $p(x_i) - p(x_{i\approx})$

14.5.2. Mean Deviation relative to the closest to inversion of N, $DM(\approx)$. Average of the absolute value of all differentials of each probability minus the closest to inversion of N, except for the differential of the closest with itself, which is excluded. Therefore, the average is taken over N minus one.

Mean Deviation relative to the closest =

$$DM(\approx) = \sum | p(x_i) - p(x_{i\approx}) | : (N - 1)$$

14.5.3. Variance relative to the probability closest to the inversion of N, $s^2(\approx)$.

Average over N minus one of all squared differentials.

Variance relative to the closest =

$$s^2(\approx) = \sum (p(x_i) - p(x_{i\approx}))^2 : (N - 1)$$

14.5.4. Standard Deviation relative to the probability closest to the inversion of N.

Square root of the Variance.

Standard Deviation relative to the closest =

$$S(\approx) = \sqrt{[\sum (p(x_i) - p(x_{i\approx}))^2 : (N - 1)]}$$

14.5.5. Critical Proportions relative to the probability closest to the inversion of N

14.5.5.1. Relative Proportions to the closest

14.5.5.1.1. Proportion of equality relative to the closest: absolute value of the relative Bias Level to the closest divided by the difference of one minus the closest.

Proportion of equality relative to the closest =

$$|p(x_i) - p(x_{i\approx})| : (1 - p(x_{i\approx}))$$

14.5.5.1.2. Proportion of positive bias relative to the closest: relative Bias Level to the closest divided by the difference of one minus the closest.

Proportion of positive bias relative to the closest =

$$(p(x_i) - p(x_{i\approx})) : (1 - p(x_{i\approx}))$$

14.5.5.1.3. Proportion of negative bias relative to the closest: closest minus empirical probability, result divided by the closest.

Proportion of negative bias relative to the closest =

$$(p(x_{i\approx}) - p(x_i)) : p(x_{i\approx})$$

14.5.5.2. Critical Proportions relative to the closest

14.5.5.2.1. Critical Proportion of equality relative to the closest. Critical probability minus the quotient of the absolute value of the relative Bias Level to the closest over the difference of one minus the closest. Zero or positive → equality to the closest accepted. Critical probability equals error percentage X divided by one hundred.

Critical Proportion of equality relative to the closest =

$p(x_c) - [| p(x_i) - p(x_{i\approx}) | : (1 - p(x_{i\approx}))] = \text{zero or positive} \rightarrow \text{equality to the closest accepted}$

$p(x_c) = X : 100$

X = error percentage

14.5.5.2.2. Critical Proportion of positive bias relative to the closest. Quotient of the relative Bias Level to the closest over the difference of one minus the closest, result minus a critical probability. Zero or positive → positive bias relative to the closest accepted. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of positive bias relative to the closest =

$$\left[\frac{p(x_i) - p(x_{i\approx})}{1 - p(x_{i\approx})} \right] - p(x_c) = \text{zero or positive} \rightarrow \text{positive bias relative to the closest accepted}$$

$p(x_c) = X : 100$

X = reliability percentage

14.5.5.2.3. Critical Proportion of negative bias relative to the closest. Closest minus empirical probability, result divided by the closest, result minus critical probability. Zero or positive → negative bias relative to the closest accepted. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of negative bias relative to the closest =

$[(p(x_i) - p(x_c)) : p(x_i)] - p(x_c)$ = zero or positive → negative bias relative to the closest accepted

$$p(x_c) = X : 100$$

X = reliability percentage

14.6. Descriptive Statistics based on a determined empirical probability “p(xn)” of the same sample in relation to the others

$p(x_n)$ = any determined empirical probability of any subject or option in the sample

14.6.1. Relative Bias Level to a determined empirical probability “p(xn)” of the same sample. Differential of each empirical probability minus that determined empirical probability “p(xn)”.

Relative Bias Level to a determined empirical probability = $p(x_i) - p(x_n)$

14.6.2. Mean Deviation relative to the determined empirical probability “ $p(x_n)$ ” of the same sample, $DM(n)$. Average of the absolute value of all differentials of each probability with respect to the determined empirical probability “ $p(x_n)$ ” of the sample, except for the differential of that same determined probability with itself.

Mean Deviation relative to a determined empirical probability =

$$DM(n) = \sum | p(x_i) - p(x_n) | : (N - 1)$$

14.6.3. Variance relative to that determined empirical probability, $s^2(n)$. Average of all squared differentials divided by N minus one.

Variance relative to a determined probability =

$$s^2(n) = \sum (p(x_i) - p(x_n))^2 : (N - 1)$$

14.6.4. Standard Deviation relative to that determined empirical probability, $S(n)$. Square root of the Variance.

Standard Deviation relative to a determined empirical probability =

$$S(n) = \sqrt{ [\sum (p(x_i) - p(x_n))^2 : (N - 1)] }$$

14.6.5. Critical Proportions relative to a determined empirical probability

14.6.5.1. Relative Proportions to a determined empirical probability

14.6.5.1.1. Proportion of equality relative to a determined empirical probability.
Absolute value of the relative Bias Level to the determined empirical probability divided by the difference of one minus the determined empirical probability.

Proportion of equality relative to a determined empirical probability =

$$| p(x_i) - p(x_n) | : (1 - p(x_n))$$

14.6.5.1.2. Proportion of positive bias relative to a determined empirical probability.
Relative Bias Level to the determined probability divided by the difference of one minus the determined empirical probability.

Proportion of positive bias relative to a determined empirical probability =

$$(p(x_i) - p(x_n)) : (1 - p(x_n))$$

14.6.5.1.3. Proportion of negative bias relative to a determined empirical probability.
Determined empirical probability minus any other, result divided by the determined probability.

Proportion of negative bias relative to a determined empirical probability =

$$(p(x_n) - p(x_i)) : p(x_n)$$

14.6.5.2. Critical Proportions relative to a determined empirical probability

14.6.5.2.1. Critical Proportion of equality relative to a determined empirical probability.

Critical probability minus the quotient of the absolute value of the relative Bias Level to the determined empirical probability over the difference of one minus the determined probability. Zero or positive → equality relative to the determined probability accepted. Critical probability equals error percentage X divided by one hundred.

Critical Proportion of equality relative to a determined empirical probability =

$p(x_c) - [| p(x_i) - p(x_n) | : (1 - p(x_n))] = \text{zero or positive} \rightarrow \text{equality relative to the determined accepted}$

$$p(x_c) = X : 100$$

X = error percentage

14.6.5.2.2. Critical Proportion of positive bias relative to a determined empirical probability.

Quotient of the Relative Bias Level to a determined empirical probability over the difference of one minus the determined probability, and the result minus the critical probability. Zero or positive → positive relative bias accepted. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of positive bias relative to a determined empirical probability =

$$\left[\frac{p(x_i) - p(x_n)}{1 - p(x_n)} \right] - p(x_c) = \text{zero or positive} \rightarrow \text{positive relative bias to the}$$

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$$p(x_c) = (X : 100)$$

X = reliability percentage

14.6.5.2.3. Critical Proportion of negative bias relative to a determined empirical probability.

Determined probability minus the empirical probability, result divided by the determined probability, and that quotient minus the critical probability. Zero or positive → negative relative bias to the determined probability accepted. Critical probability equals reliability percentage X divided by one hundred.

Critical Proportion of negative bias relative to a determined empirical probability =

$[(p(x_n) - p(x_i)) : p(x_n)] - p(x_c) = \text{zero or positive} \rightarrow \text{negative relative bias accepted}$

$$p(x_c) = (X : 100)$$

X = reliability percentage

15. Rational Critique of the Standard Score in the Second Method

15.1. The Standard Score in the First Method, classical or conventional statistics

15.1.1. The Standard Score equals the quotient of the differential score over the Standard Deviation.

15.1.2. The differential score equals the difference between the raw score and the arithmetic mean.

15.1.3. The Standard Deviation equals the square root of the Variance.

15.1.4. The Variance equals the average of the sum of squared differential scores.

15.1.5. The sum of differential scores without squaring—if averaged as absolute values—equals the Mean Deviation.

15.1.6. The basis of every dispersion statistic is the differential score, whose average

of summed absolute values is the Mean Deviation; whose average of summed squares is the Variance; whose square root is the Standard Deviation.

15.1.7. The Standard Score is the contrast—via quotient—of the differential score over the Standard Deviation which, under normal conditions (insofar as all subjects or options tend to be normal), necessarily yields the greatest concentration of Standard Scores between +1 and –1 on the Gaussian bell curve.

15.1.8. A Standard Score whose quotient lies between +1 and –1 means the absolute value of the differential score is equal to or less than the Standard Deviation. If the Standard Score equals +1 or –1, then the absolute value of the differential score equals the Standard Deviation. The quotient will be positive or negative depending on the sign of the differential.

15.1.9. A Standard Score greater than +1 or less than –1 means the absolute value of the differential score exceeds the Standard Deviation.

15.1.10. A negative Standard Score implies the raw score is below the arithmetic mean; if, in absolute terms, the negative differential exceeds the Standard Deviation, then the Standard Score will be less than –1.

15.1.11. A negative Standard Score, insofar as the raw score is below the arithmetic mean but the absolute value of the negative differential does not exceed the Standard Deviation, will be a negative Standard Score between 0 and –1.

15.1.12. A raw score above the arithmetic mean yields a positive differential score; the more positive it is and the more it exceeds the Standard Deviation, the more the Standard Score exceeds +1.

15.1.13. A raw score above the arithmetic mean, but whose positive differential does not exceed the Standard Deviation, yields a positive Standard Score between 0 and +1.

15.1.14. The Standard Score of the First Method is, in essence, a contrast between individual differential score and Standard Deviation—that is, between individual empirical dispersion and sample empirical dispersion obtained from the Standard Deviation.

15.2. The Rational Critique of the normal Standard Score

15.2.1. The traditional Gauss-type critique

15.2.1.1. The First Method (traditional statistics) normally contrasts Standard Score values using critical values—critical integral area percentages—obtained from the Gaussian bell curve.

15.2.1.2. The Gaussian bell's critical values for rationally critiquing Standard Scores are geometric values; they measure proportional area against which Standard Scores are contrasted.

15.2.1.3. The traditional critique of the First Method concerning the Standard Score is a geometrically integral critique.

15.2.2. The Second Method's critique of the Standard Score reformulated by *Impossible Probability*

15.2.2.1. *Impossible Probability* is, in essence, a reformulation of traditional mathematics. What traditional mathematics calls the arithmetic mean, *Impossible Probability* calls the inversion of N—that is, the theoretical probability of occurrence under equality of opportunities. What traditional mathematics calls the differential score, *Impossible Probability* calls the normal Bias Level, as it normally measures the bias of the phenomenon under study. What traditional mathematics calls relative frequency, *Impossible Probability* transforms into empirical probability: a model of statistical probability—i.e., the statistics of probability. In short, the Second Method tends to be an alternative method for the study of statistics. *Impossible Probability* is an essential redefinition—a reformulation of the general principles of elementary statistics. It starts from new meanings for traditional concepts such as central tendency or dispersion. *Impossible Probability* is a redefinition or reformulation of the foundations of statistics and probability toward a statistical probability or statistics of probability.

15.2.2.2. Within this reformulation of the most elementary mathematical concepts, *Impossible Probability* necessarily reformulates the Standard Score, which, in essence from Gauss to the present, is the quotient contrast of differential score over Standard Deviation; in terms of the Second Method, the contrast of the normal Bias Level over the central tendency of the normal bias.

15.2.2.3. If Gauss's critique of the Standard Score is integral and geometric in nature, then *Impossible Probability's* critique must necessarily be rational. Here lies the

greatest difference between Gauss's criticism and the Second Method's critical rationalism of the Standard Score: for the Second Method, the Standard Score must be critiqued rationally.

15.2.2.4. Every rational critique must above all be a critique grounded in sufficient mathematical reasons to criticize the tendency and to determine when a tendency is sufficiently rational to be provisionally accepted as true.

15.2.2.5. The critique of the normal Standard Score, in any of its forms within the Second Method, must therefore be a rational critique.

15.2.2.6. Within the statistical tests of rational critique of the normal Standard Score for positive bias, two sub-models must be distinguished: (a) those statistical tests for studying positive bias under models of Maximum Theoretical Possible Bias, Maximum Theoretical Possible Mean Deviation, or Maximum Theoretical Possible Standard Deviation—for rational critique under the hypothesis that, out of N , only a single subject or option is ideal; and (b) other models for studying positive bias in samples with more than one subject or option comprising an omega ideal subset greater than one and less than N . An "ideal subset" is formed by those subjects or options defined as ideal when there is more than one ideal subject or option simultaneously in the sample, with a minimum of two and a maximum of N minus one ideal subjects or options—that is, the magnitude of ideals varies between two and N minus one. Logically, if in a given N there were a single subject or option whose ideal were to increase negative bias, or if out of the entire N only a single subject or option were non-ideal, the way to study that single subject's or option's negative bias would be by studying N insofar as the magnitude of ideals would be N minus one.

15.2.2.7. Accordingly, the exposition will begin with the more standard cases—those equations for critiquing a given reality under equality of opportunities or when studying bias under the hypothesis of maximum possible dispersion where only a single subject or option is ideal—and will conclude this section by presenting developments for omega conditions.

15.2.3. Rational Critique of the normal Standard Score

15.2.3.1. The models of rational critique of the normal Standard Score will be valid for any universe of infinite subjects or options, but not applicable to every universe of

limited (finite) options, and in no case applicable to universes limited to two options. In a three-option model it may likewise present certain limitations in the Rational Critique of the Standard Score when the object of study comprises only a single ideal subject or option; thus, if the object is to increase empirical dispersion to the maximum possible, it would be more advisable to apply the Rational Critique of the normal Standard Score from four options upward.

15.2.3.2. Depending on the object of study, an ideal critical ratio will be established for the rational critique of normal Standard Scores, to accept or reject an observed tendency.

15.2.3.3. The tendency depends on the object of study.

15.2.3.4. In summary, the object of study may be: equality of opportunities, study of positive bias, or study of negative bias. According to the object of study, the ideal critical ratio—the critical probability—is determined.

15.2.3.5. Thus, the rational critique of the normal Standard Score in *Impossible Probability* will depend on the object of study, insofar as it rationally critiques whether a tendency sufficiently approaches the ideal.

15.2.3.6. Normally, at a normal Bias Level, one says that a subject or option tends toward equality of opportunities if the normal Bias Level tends to zero. Hence, a normal Bias Level tending to zero, divided by the normal Mean or Standard Deviation, should be a quotient that tends to zero; therefore the rational critique must assess whether the tendency of the normal Standard Score toward zero is sufficient to guarantee equality of opportunities.

15.2.3.7. Normally, a positive normal Bias Level will tend to be an empirical probability greater than the inversion of N; thus, the greater the positive differential, the larger the positive value of the normal Standard Score. The rational critique must then determine whether the positive increase of the Standard Score is sufficient to determine an appropriate positive bias.

15.2.3.8. Normally, in the study of normal negative bias, the Bias Level will tend to be less than the inversion of N; thus, as the normal Standard Score—(inversion of N –

empirical probability) relative to the sample dispersion—grows, negative bias will be increasingly guaranteed.

15.2.4. Rational Critique of the normal Standard Score in *Impossible Probability*

15.2.4.1. Types of normal Standard Score

15.2.4.1.1. The **Standard Score of Equality**, for rational critique in studies of equality of opportunities, equals the absolute value of the normal Bias Level divided by the normal Mean or Standard Deviation (DMoS). According to the scientific model's choice of empirical dispersion (Mean Deviation or Standard Deviation), one or the other is used in the denominator. It is "normal" Mean or Standard Deviation according to whether one estimates the average of the sum of absolute values (Mean Deviation) or the square root of the sum of squares (Standard Deviation) of the normal Bias Levels, with the normal Bias Level being the difference of empirical probability minus inversion of N, $1/N$.

15.2.4.1.2. The **Standard Score of Positive Bias**, for rational critique in studies of positive bias, equals the normal Bias Level divided by the normal Mean or Standard Deviation, according to whether the scientific model accepts Mean Deviation or Standard Deviation.

15.2.4.1.3. The **Standard Score of Negative Bias**, for rational critique in studies of negative bias, equals the difference of inversion of N minus the empirical probability; this differential is then divided by the normal Mean or Standard Deviation, as decided by scientific policy.

15.2.4.2. Rational Critique of the normal Standard Score

15.2.4.2.1. Rational Critique of the Standard Score using Mean Deviation

15.2.4.2.1.1. Rational Critique of the Standard Score of Equality, using Mean Deviation, equals the difference of the critical probability minus the result of dividing the absolute value of the normal Bias Level by the normal Mean Deviation. A zero or positive result → the tendency toward equality of opportunities is rationally accepted. The critical probability equals the product of the error percentage X, divided by one hundred, by the quotient of Maximum Theoretical Possible Bias over Maximum Theoretical Possible Mean Deviation.

Rational Critique of the Standard Score of Equality =

$p(x_c) - [| p(x_i) - 1/N | : DM] = \text{zero or positive} \rightarrow \text{equality of opportunities accepted}$

DM = Mean Deviation

$p(x_c) = \{ (1 - 1/N) : [(1 - 1/N) \cdot 2] : N \} \cdot (X : 100)$

X = error percentage

15.2.4.2.1.2. Rational Critique of the Standard Score of Positive Bias, using Mean Deviation: quotient of the normal Bias Level by the normal Mean Deviation, minus a critical probability. Zero or positive → sufficient tendency to positive bias accepted. The critical probability equals the reliability percentage X, divided by one hundred, times the quotient of Maximum Theoretical Possible Bias over Maximum Theoretical Possible Mean Deviation.

Rational Critique of the Standard Score of Positive Bias =

$[(p(x_i) - 1/N) : DM] - p(x_c) = \text{zero or positive} \rightarrow \text{positive bias accepted}$

DM = Mean Deviation

$p(x_c) = \{ (1 - 1/N) : \{ [(1 - 1/N) \cdot 2] : N \} \} \cdot (X : 100)$

X = reliability percentage

15.2.4.2.1.3. Rational Critique of the Standard Score of Negative Bias, using Mean Deviation: difference of inversion of N minus empirical probability, the result divided by the normal Mean Deviation; then subtract a critical probability. Zero or positive → sufficient rational tendency to negative bias accepted. The critical probability equals the product of the reliability percentage X, divided by one hundred, by the quotient of inversion of N over Maximum Theoretical Possible Mean Deviation.

Rational Critique of the Standard Score of Negative Bias =

$[(1/N - p(xi)) : DM] - p(xc) = \text{zero or positive} \rightarrow \text{negative bias accepted}$

DM = Mean Deviation

$p(xc) = \{ 1/N : \{ [(1 - 1/N) \cdot 2] : N \} \} \cdot (X : 100)$

X = reliability percentage

15.2.4.2.2. Rational Critique of the normal Standard Score using Standard Deviation

15.2.4.2.2.1. Rational Critique of the Standard Score of Equality in the study of equality of opportunities, using Standard Deviation, equals the difference of a critical probability minus the result of dividing the absolute value of the normal Bias Level by the normal Standard Deviation. Zero or positive → sufficient tendency to equality of opportunities rationally accepted. The critical probability equals the product of the error percentage X, divided by one hundred, by the quotient of Maximum Theoretical Possible Bias over Maximum Theoretical Possible Standard Deviation.

Rational Critique of the Standard Score of Equality =

$p(x_c) - [| p(x_i) - 1/N | : S] = \text{zero or positive} \rightarrow \text{equality of opportunities accepted}$

S= Standard Deviation

$$p(x_c) = \{ (1 - 1/N) : \sqrt{ [(1 - 1/N)^2 + ((1/N)^2 \cdot (N - 1))] : N } \} \cdot (X : 100)$$

X = error percentage

15.2.4.2.2. Rational Critique of the Standard Score of Positive Bias, using Standard Deviation: normal Bias Level over normal Standard Deviation, minus a critical probability. Zero or positive → sufficient tendency to positive bias rationally accepted. The critical probability equals the reliability percentage X, divided by one hundred, times the quotient of Maximum Theoretical Possible Bias over Maximum Theoretical Possible Standard Deviation.

Rational Critique of the Standard Score of Positive Bias =
 $[(p(x_i) - 1/N) : S] - p(x_c) = \text{zero or positive} \rightarrow \text{positive bias accepted}$

S= Standard Deviation

$$p(x_c) = \{ (1 - 1/N) : \sqrt{ [(1 - 1/N)^2 + ((1/N)^2 \cdot (N - 1))] : N } \} \cdot (X : 100)$$

X = reliability percentage

15.2.4.2.2.3. Rational Critique of the Standard Score of Negative Bias, using Standard Deviation: difference of inversion of N minus empirical probability, the result divided by the normal Standard Deviation, and from that quotient subtract a critical probability. Zero or positive → negative bias accepted. The critical probability equals the reliability percentage X, divided by one hundred, times the quotient of inversion of N over Maximum Theoretical Possible Standard Deviation.

Rational Critique of the Standard Score of Negative Bias =

$[(1/N - p(x_i)) : S] - p(x_c) = \text{zero or positive} \rightarrow \text{negative bias accepted}$

S= Standard Deviation

$$p(x_c) = \{ 1/N : \sqrt{[(1 - 1/N)^2 + ((1/N)^2 \cdot (N - 1))]} : N \} \cdot (X : 100)$$

X = reliability percentage

15.3. Critical Proportions of the normal Standard Score

15.3.1. Concept of Critical Proportions of the normal Standard Score

15.3.1.1. A proportion is the quotient of something over its maximum tendency.

15.3.1.2. If one can rationally critique any given Standard Score, it is because one can know *a priori* what its maximum tendency is.

15.3.1.3. Regarding the maximum tendency of a Standard Score, in an error study: error percentage X, divided by one hundred, times the maximum tendency equals the critical

probability as a margin of error. A Standard Score equal to or less than the critical probability is accepted in an error study.

15.3.1.4. Regarding the maximum tendency of a Standard Score, in a reliability study: reliability percentage X, divided by one hundred, times the maximum tendency equals the critical probability as a margin of reliability. A Standard Score equal to or greater than the critical probability is accepted.

15.3.1.5. If, in order to rationally critique a Standard Score, it is necessary to know *a priori* what the maximum possible tendency is, then: in an error study, based on the **minimum error percentage** over the maximum possible tendency, only that Standard Score **equal to or less than** the minimum possible error is accepted; or, in a reliability study, based on the **maximum reliability percentage** over the maximum possible tendency, only that Standard Score **equal to or greater than** the maximum possible reliability is accepted.

15.3.1.6. Thus, if any model of rational critique of any Standard Score model is established upon *a priori* knowledge of the maximum tendency—so that maximum tendency times percentage X (error or reliability), divided by one hundred, equals the error or reliability margin one is willing to accept.

15.3.1.7. If the maximum possible tendency is known *a priori*, then a simpler way to critique the Standard Score is simply by the quotient of the Standard Score over its maximum tendency; thus, the quotient “Standard Score / maximum tendency” equals the **proportion** of the maximum tendency represented by the Standard Score.

15.3.1.8. In an **error** study, any quotient “Standard Score / maximum tendency” whose result is **equal to or less than** the error percentage X divided by one hundred (**Table 1**) is accepted directly.

15.3.1.9. In a **reliability** study, any quotient “Standard Score / maximum tendency” whose result is **equal to or greater than** the reliability percentage X divided by one hundred (**Table 1**) is accepted directly.

15.3.2. Critical Proportion of the normal Standard Score

15.3.2.1. Critical Proportion of the Standard Score, using Mean Deviation

15.3.2.1.1. Critical Proportion of the Standard Score of Equality using Mean Deviation

15.3.2.1.1.1. Proportion of the Standard Score of Equality using Mean Deviation: one minus the quotient of the Standard Score of Equality (absolute value of the Bias Level over Mean Deviation) over the quotient of Maximum Theoretical Possible Bias by Maximum Theoretical Possible Mean Deviation.

Proportion of the Standard Score of Equality =

$$1 - \{ [|p(x_i) - 1/N| : DM] : \{ (1 - 1/N) : \{ [(1 - 1/N) \cdot 2] : N \} \} \}$$

DM= Mean Deviation

15.3.2.1.1.2. Critical Proportion of the Standard Score of Equality using Mean Deviation: critical probability minus the Proportion of the Standard Score of Equality; zero or positive → accepted. Critical probability equals the reliability percentage X divided by one hundred (**Table 1**).

Critical Proportion of the Standard Score of Equality =
 $\{ 1 - \{ [|p(x_i) - 1/N| : DM] : \{ (1 - 1/N) : \{ [(1 - 1/N) \cdot 2] : N \} \} \} \} - p(x_c) = \text{zero or positive}$
→ equality accepted

$$p(x_c) = \frac{X}{100}$$

X = reliability percentage

15.3.2.1.1.3. Percentage of the Standard Score of Equality using Mean Deviation: one hundred times the Proportion of the Standard Score of Equality.

Percentage of the Standard Score of Equality =

$$\{1 - \{ [|p(x_i) - 1/N| : DM] : \{ (1 - 1/N) : \{ [(1 - 1/N) \cdot 2] : N \} \} \} \} \cdot 100$$

DM= Mean Deviation

15.3.2.1.1.4. Critical Percentage of the Standard Score of Equality using Mean Deviation: Percentage of the Standard Score of Equality minus the reliability percentage X; zero or positive → accepted.

Critical Percentage of the Standard Score of Equality =

$$\{ \{ 1 - \{ [|p(x_i) - 1/N| : DM] : \{ (1 - 1/N) : \{ [(1 - 1/N) \cdot 2] : N \} \} \} \} \cdot 100 \} - X = \text{zero or positive} \rightarrow \text{equality accepted}$$

DM= Mean Deviation

X = reliability percentage

15.3.2.1.2. Critical Proportion of the Standard Score of Positive Bias using Mean Deviation

15.3.2.1.2.1. Proportion of the Standard Score of Positive Bias using Mean Deviation: Standard Score of Positive Bias (Bias Level over Mean Deviation) divided by the quotient of Maximum Theoretical Possible Bias by Maximum Theoretical Possible Mean Deviation.

Proportion of the Standard Score of Positive Bias =

$$[(p(x_i) - 1/N) : DM] : \{ (1 - 1/N) : \{ [(1 - 1/N) \cdot 2] : N \} \}$$

DM= Mean Deviation

15.3.2.1.2.2. Critical Proportion of the Standard Score of Positive Bias using Mean Deviation: Proportion of the Standard Score of Positive Bias minus the critical probability; zero or positive → accepted. Critical probability equals the reliability percentage X divided by one hundred (**Table 1**).

Critical Proportion of the Standard Score of Positive Bias =

$$\{ [(p(x_i) - 1/N) : DM] : \{ (1 - 1/N) : \{ [(1 - 1/N) \cdot 2] : N \} \} \} - p(x_c) = \text{zero or positive} \rightarrow$$

positive bias accepted

$$p(x_c) = X : 100$$

X = reliability percentage

15.3.2.1.2.3. Percentage of the Standard Score of Positive Bias using Mean Deviation: one hundred times the Proportion of the Standard Score of Positive Bias.

Percentage of the Standard Score of Positive Bias =

$$\{[(p(x_i) - 1/N) : DM] : \{(1 - 1/N) : [(1 - 1/N) \cdot 2] : N\}\} \cdot 100$$

DM= Mean Deviation

15.3.2.1.2.4. Critical Percentage of the Standard Score of Positive Bias using Mean Deviation: Percentage of the Standard Score of Positive Bias minus the reliability percentage X; zero or positive → positive bias accepted.

Critical Percentage of the Standard Score of Positive Bias =

$$\{[(p(x_i) - 1/N) : DM] : \{(1 - 1/N) : [(1 - 1/N) \cdot 2] : N\}\} \cdot 100\} - X = \text{zero or positive} \\ \rightarrow \text{positive bias accepted}$$

DM= Mean Deviation

X = reliability percentage

15.3.2.1.3. Critical Proportion of the Standard Score of Negative Bias using Mean Deviation

15.3.2.1.3.1. Proportion of the Standard Score of Negative Bias using Mean Deviation: Standard Score of Negative Bias (difference of inversion of N minus empirical probability, result over Mean Deviation) divided by the quotient of inversion of N by Maximum Theoretical Possible Mean Deviation.

Proportion of the Standard Score of Negative Bias =

$$[(1/N - p(x_i)) : DM] : \{1/N : [(1 - 1/N) \cdot 2] : N\}$$

DM= Mean Deviation

15.3.2.1.3.2. Critical Proportion of the Standard Score of Negative Bias using Mean Deviation: Proportion of the Standard Score of Negative Bias minus the critical probability; zero or positive → accepted. Critical probability equals the reliability percentage X divided by one hundred (**Table 1**).

Critical Proportion of the Standard Score of Negative Bias =

$$\{[(1/N - p(x_i)) : DM] : \{1/N : [(1 - 1/N) \cdot 2] : N\}\} - p(x_c) = \begin{matrix} \text{zero or positive} \rightarrow \text{negative} \\ \text{bias} \hspace{15em} \text{accepted} \end{matrix}$$

$$p(x_c) = \frac{X}{100}$$

X = reliability percentage

15.3.2.1.3.3. Percentage of the Standard Score of Negative Bias using Mean

Deviation: one hundred times the Proportion of the Standard Score of Negative Bias.

Percentage of the Standard Score of Negative Bias =

$$\{[(1/N - p(x_i)) : DM] : \{1/N : [(1 - 1/N) \cdot 2] : N\}\} \cdot 100$$

DM= Mean Deviation

15.3.2.1.3.4. Critical Percentage of the Standard Score of Negative Bias using Mean

Deviation: Percentage of the Standard Score of Negative Bias minus the reliability percentage X; zero or positive → accepted.

Critical Percentage of the Standard Score of Negative Bias =

$$\{[(1/N - p(x_i)) : DM] : \{1/N : [(1 - 1/N) \cdot 2] : N\}\} \cdot 100\} - X = \begin{matrix} \text{negative} & & \text{bias} & & \text{accepted} \end{matrix}$$

DM= Mean Deviation

X = reliability percentage

15.3.2.2. Critical Proportion of the Standard Score, using Standard Deviation, S

15.3.2.2.1. Critical Proportion of the Standard Score of Equality using Standard Deviation

15.3.2.2.1.1. Proportion of the Standard Score of Equality using Standard Deviation (S): one minus the quotient of the Standard Score of Equality (absolute value of the Bias Level over Standard Deviation) over the quotient of Maximum Theoretical Possible Bias by Maximum Theoretical Possible Standard Deviation.

Proportion of the Standard Score of Equality =

$$1 - \{ [| p(x_i) - 1/N | : S] : \{ (1 - 1/N) : \sqrt{ [(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]] : N } \} \}$$

S= Standard Deviation

15.3.2.2.1.2. Critical Proportion of the Standard Score of Equality using Standard Deviation

Proportion of the Standard Score of Equality minus the critical probability; zero or positive → accepted. Critical probability equals the reliability percentage X divided by one hundred (Table 1).

Critical Proportion of the Standard Score of Equality =

$$\{ 1 - \{ [| p(x_i) - 1/N | : S] : \{ (1 - 1/N) : \sqrt{ [(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]] : N } \} \} \} - p(x_c)$$

= zero or positive → equality accepted

$$p(x_c) = X : 100$$

X = reliability percentage

15.3.2.2.1.3. Percentage of the Standard Score of Equality using Standard Deviation

Product of one hundred by the Proportion of the Standard Score of Equality.

Percentage of the Standard Score of Equality =

$$\{1 - \{[|p(x_i) - 1/N| : S] : \{(1 - 1/N) : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]] : N}\}\}\} \cdot 100$$

S= Standard Deviation

15.3.2.2.1.4. Critical Percentage of the Standard Score of Equality using Standard Deviation

Percentage of the Standard Score of Equality minus the reliability percentage X; zero or positive → equality accepted.

Critical Percentage of the Standard Score of Equality =

$$\{\{1 - \{[|p(x_i) - 1/N| : S] : \{(1 - 1/N) : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]] : N}\}\}\} \cdot 100\} - X =$$

zero or positive → equality accepted

S= Standard Deviation

X = reliability percentage

15.3.2.2.2. Critical Proportion of the Standard Score of Positive Bias using Standard Deviation

15.3.2.2.2.1. Proportion of the Standard Score of Positive Bias using Standard Deviation

Standard Score of Positive Bias (Bias Level over Standard Deviation), divided by the quotient of Maximum Theoretical Possible Bias by Maximum Theoretical Possible Standard Deviation.

Proportion of the Standard Score of Positive Bias =

$$[(p(x_i) - 1/N) : S] : \{(1 - 1/N) : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]]} : N\}$$

S= Standard Deviation

15.3.2.2.2.2. Critical Proportion of the Standard Score of Positive Bias using Standard Deviation

Proportion of the Standard Score of Positive Bias minus the critical probability; zero or positive → accepted. Critical probability equals the reliability percentage X divided by one hundred (Table 1).

Critical Proportion of the Standard Score of Positive Bias =

$$\{[(p(x_i) - 1/N) : S] : \{(1 - 1/N) : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]]} : N\}\} - p(x_c) = \text{zero or positive} \rightarrow \text{positive bias accepted}$$

S= Standard Deviation

$$p(x_c) = X : 100$$

X = reliability percentage

15.3.2.2.3. Percentage of the Standard Score of Positive Bias using Standard Deviation

Product of one hundred by the Proportion of the Standard Score of Positive Bias.

Percentage of the Standard Score of Positive Bias =

$$\{[(p(x_i) - 1/N) : S] : \{(1 - 1/N) : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]]} : N\}\} \cdot 100$$

S= Standard Deviation

15.3.2.2.4. Critical Percentage of the Standard Score of Positive Bias using Standard Deviation

Percentage of the Standard Score of Positive Bias minus the reliability percentage X;
zero or positive → positive bias accepted.

Critical Percentage of the Standard Score of Positive Bias =

$$\{[(p(x_i) - 1/N) : S] : \{(1 - 1/N) : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]]} : N\}\} \cdot 100\} - X =$$

zero or positive → positive bias accepted

S= Standard Deviation

X = reliability percentage

15.3.2.2.3. Critical Proportion of the Standard Score of Negative Bias using Standard Deviation

15.3.2.2.3.1. Proportion of the Standard Score of Negative Bias using Standard Deviation

Standard Score of Negative Bias (difference of inversion of N minus empirical probability, result over Standard Deviation), divided by the quotient of inversion of N by Maximum Theoretical Possible Standard Deviation.

Proportion of the Standard Score of Negative Bias =

$$[(1/N - p(x_i)) : S] : \{1/N : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]] : N}\}$$

S= Standard Deviation

15.3.2.2.3.2. Critical Proportion of the Standard Score of Negative Bias using Standard Deviation

Proportion of the Standard Score of Negative Bias minus the critical probability; zero or positive → accepted. Critical probability equals the reliability percentage X divided by one hundred (Table 1).

Critical Proportion of the Standard Score of Negative Bias =

$$\{[(1/N - p(x_i)) : S] : \{1/N : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]] : N}\}\} - p(x_c) = \text{zero or positive} \rightarrow \text{negative bias accepted}$$

S= Standard Deviation

$$p(x_c) = X : 100$$

X = reliability percentage

15.3.2.2.3.3. Percentage of the Standard Score of Negative Bias using Standard Deviation

Product of one hundred by the Proportion of the Standard Score of Negative Bias.

Percentage of the Standard Score of Negative Bias =

$$\{[(1/N - p(x_i)) : S] : \{1/N : \sqrt{[(1 - 1/N)^2 + [(1/N)^2 \cdot (N - 1)]] : N}\}\} \cdot 100$$

S= Standard Deviation

15.3.2.2.3.4. Critical Percentage of the Standard Score of Negative Bias using Standard Deviation

Percentage of the Standard Score of Negative Bias minus the reliability percentage X; zero or positive → accepted.

Critical Percentage of the Standard Score of Negative Bias =

$$\{[(1/N - p(x_i)) : S] : \{1/N : \sqrt{[(1 - 1/N)^2 + [1/N^2 \cdot (N - 1)]] : N}\}\} \cdot 100\} - X = \text{zero or positive} \rightarrow \text{negative bias accepted}$$

S= Standard Deviation

X = reliability percentage

15.4. Rational Critique of the Standard Score under omega conditions

15.4.1. The omega conditions

15.4.1.1. If, in any kind of universe with N greater than two, there exists a subset of ideals between two and N minus one, inclusive, then omega conditions are said to hold.

15.4.1.2. The critique of reality under omega conditions must be different from the critique of reality under non-omega conditions.

15.4.1.3. Non-omega conditions are the following: either all subjects or options are equal in importance to each other (equality of opportunities), or there are conditions of tendency toward maximum possible dispersion, or the ideal is zero non-ideal qualities (the sample of zeros).

15.4.1.4. In any kind of universe, conditions of maximum dispersion are said to occur when there is only a single ideal in N .

15.4.1.5. If in N there is only a single ideal, then the more the empirical value of the ideal subject or option increases, the more dispersion tends toward the maximum possible dispersion; that is, the greater the empirical probability of the ideal subject or option, the more the Bias Level tends to the Maximum Theoretical Possible Bias, and the Mean or Standard Deviation tends to the Maximum Theoretical Possible Mean or Standard Deviation.

15.4.1.6. Under omega conditions—i.e., in any universe with N greater than two where there exists an omega subset of ideals between two and N minus one, both limits included—if all the ideals are ideal to the same magnitude (hence all ideals equally tend to share the same maximum empirical probability, $p(x_i+)$), then the dispersion conditions, whether individual or sample, will be further from the maximum possible the larger omega becomes.

15.4.1.7. Dispersion is a variable dependent on three factors: **N** , the raw scores or frequencies, and the magnitude of ideals within **N** . The greater the magnitude of ideals in **N** , the more dispersion will tend to differ from the maximum possible; thus, if the entire **N** is equally ideal, the model will tend toward **zero dispersion** due to equality of opportunities. Conversely, the smaller the magnitude of ideals in **N** , the more

dispersion will tend to be identical to the maximum. Conditions of **maximum possible dispersion** occur only when, out of the entire **N**, there is **only one** possible ideal and, moreover, that single ideal has the greatest possible empirical value—**empirical probability equal to one**.

15.4.2. Models of Rational Critique of the Standard Score of positive bias under omega conditions

15.4.2.1. Logically, the models of rational critique must be different under **omega conditions**, i.e., for an **N > 2** with a subset of ideals between **2** and **N – 1**.

15.4.2.2. In omega models, necessarily—since the study centers on the behavior of the ideals within the omega subset—the analysis focuses on the **tendency of the positive bias of the ideals**.

15.4.2.3. Thus, what must be critiqued is the **Standard Score of positive bias**, which equals the **Bias Level** divided by either **Mean Deviation** or **Standard Deviation**, depending on whether the study uses Mean Deviation or Standard Deviation (DMoS).

15.4.2.4. Accordingly, if under the hypothesis of **maximum possible dispersion** one contrasted whether the Standard Score of Positive Bias was **equal to or greater than a critical value**—that critical value being the **reliability percentage X**, divided by one hundred, times the quotient of **Maximum Theoretical Possible Bias** over **Maximum Theoretical Possible Mean or Standard Deviation**—

15.4.2.5. Under omega conditions, one will critique the result of **Bias Level over Mean or Standard Deviation** relative to a **critical value** equal to the **reliability percentage X**, divided by one hundred, times the quotient of the difference (**inversion of omega minus inversion of N**) over the **Ideal Mean or Standard Deviation** (the “Ideal” dispersion being that which would result if, for all omega values, the empirical probability were **inversion of omega**).

15.4.2.5.1. Rational Critique of the Omega Standard Score using Mean Deviation, DM.

Bias Level over normal Mean Deviation, result minus the critical probability. Zero or positive → accepted. Critical probability equals the product of the reliability percentage

X, divided by one hundred, by the quotient of (**inversion of omega minus inversion of N**) divided by **Ideal Mean Deviation**.

Rational Critique of the Omega Standard Score (DM) =

$[(p(x_i) - 1/N) : DM] - p(x_c) = \text{zero or positive} \rightarrow \text{positive bias accepted}$

MD= Mean Deviation

$$p(x_c) = \{ (1/\Omega - 1/N) : \{ [(1/\Omega - 1/N) \cdot \Omega] + [1/N \cdot (N - \Omega)] : N \} \} \cdot (X : 100)$$

X = reliability percentage

15.4.2.5.2. Rational Critique of the Omega Standard Score using Standard Deviation, S.

Bias Level over normal Standard Deviation, result minus the critical probability. Zero or positive $\rightarrow$ accepted. Critical probability equals the product of the reliability percentage **X**, divided by one hundred, by the quotient of (**inversion of omega minus inversion of N**) divided by **Ideal Standard Deviation**.

Rational Critique of the Omega Standard Score (S) =

$[(p(x_i) - 1/N) : S] - p(x_c) = \text{zero or positive} \rightarrow \text{positive bias accepted}$

$$p(x_c) = \{ (1/\Omega - 1/N) : \sqrt{ \{ [(1/\Omega - 1/N)^2 \cdot \Omega] + [(1/N)^2 \cdot (N - \Omega)] \} : N \} \} \cdot (X : 100)$$

S= Standard Deviation

X = reliability percentage

15.4.3. Critical Ideal Similarity of the Standard Score

15.4.3.1. Under normal conditions, a given proportion studies the proportion of a part relative to a whole or to a maximum value.

15.4.3.2. The reason why it is feasible to compute **Critical Proportions of the Standard Score** under normal conditions—whether under equality of opportunities or under the hypothesis of maximum possible dispersion—is that, if one can compute a certain **maximum possible dispersion**, then, given a particular dispersion, one can compute the **proportion** between that particular dispersion and the maximum possible.

15.4.3.3. In the case of omega conditions, the value of the **Ideal Mean Deviation** or **Ideal Standard Deviation** is **not a maximum value**; it is simply **ideal**, but not maximal.

15.4.3.4. Since, in a given study under omega conditions, there may be situations where **non-ideal** subjects or options can even reach an empirical probability **equal to or greater than** the inversion of **N**, or where **ideal** subjects or options fall **below** inversion of **omega**—possibly much lower than an acceptable error margin—while some ideal subjects within omega may be **close to an empirical probability of one**: these are conditions in which, although the study begins *a priori* focusing on an omega subset, the **individual or sample dispersion** need **not** necessarily adjust to the ideal.

15.4.3.5. Since, for these studies, the ideal is **not** a maximum dispersion but an **ideal dispersion**—defining ideal dispersion such that, for each **ideal** subject or option, the difference (**empirical probability – inversion of omega**) tends to zero, and at the sample level the necessary conditions hold to sufficiently tend toward **Ideal Mean Deviation** or **Ideal Standard Deviation** (ideal **under omega conditions**, hence not necessarily maximal values)—then, the larger **omega** becomes, the greater the difference between empirical (individual or sample) dispersion and maximal (individual or sample) dispersion. Therefore, **any proportionality study** between empirical and ideal values in an **omega** setting **lacks logic**: omega-ideal values are **not maximal ideals**, but ideals **given omega conditions**.

15.4.3.6. Since, in an omega study, the object of study regarding **quotients of real vs. ideal values** should focus less on proportionality and more on how **real values adjust**

to the ideal, one way to do this in *Impossible Probability* is via **similarity**, defined as the **absolute value of the base-10 logarithm** of (real value divided by ideal value). The closer this logarithm tends to **zero**, the closer the quotient tends to **one**, hence the greater the **identity** between ideal and real values—within a reliability margin. Acceptance requires that this log-quotient be **equal to or less than** a reliability margin equal to the **absolute value of the base-10 logarithm** of (reliability percentage X divided by one hundred).

15.4.3.7. In general, the **Critical Ideal Similarity of the Standard Score** equals the **absolute value of the base-10 logarithm** of the quotient of the **Standard Score of Positive Bias** divided by ((inversion of omega – inversion of N) / Ideal Mean or Ideal Standard Deviation), depending on whether the study is conducted with Mean Deviation or Standard Deviation. The logarithm's result must be **equal to or less than** the **absolute value of the base-10 logarithm** of (X : 100).

15.4.3.7.1. Critical Ideal Similarity of the Standard Score using Ideal Mean Deviation =

$$p(xc) - \{ | \text{Log} \{ [(p(xi) - 1/N) : DM] : \{ (1/\Omega - 1/N) : \{ [(1/\Omega - 1/N) \cdot \Omega] + [1/N \cdot (N - \Omega)] : N \} \} \} \} \}$$

= zero or positive → positive bias accepted

DM=Mean Deviation

$$p(xc) = | \text{Log} (X : 100) |$$

X = reliability percentage

15.4.3.7.2. Critical Ideal Similarity of the Standard Score using Ideal Standard

Deviation =

$$p(xc) - \{ | \text{Log} \{ [(p(xi) - 1/N) : S] : \{ (1/\Omega - 1/N) : \sqrt{\{ [(1/\Omega - 1/N)^2 \cdot \Omega] + [(1/N)^2 \cdot (N - \Omega)] \}} : N \} \} \} | \}$$

= zero or positive → positive bias accepted

S= Standard Deviation

$$p(xc) = | \text{Log} (X : 100) |$$

X = reliability percentage

15.5. Final assessments

15.5.1. To conclude—though it was previously noted—**all** models of rational critique of **normal Standard Scores** may be applied universally to any **infinite** or **finite** universe, **except** to models limited to **two options**. The reason it is illogical to apply the rational critique of normal Standard Scores to two-option models is simple: the **normal Standard Score** in **any** two-option model will **universally be one**; therefore applying this model to two options lacks logic. Likewise, this model may have certain limitations in **three-option** models when the object of study is **maximum possible dispersion**; under this object of study, **Rational Critique of Standard Scores** is advisable from **four options** or more.

15.5.2. In any study, depending on the **approach**, **type of universe**, **object of study**, and **ideals**, scientific policy must design the model coherently with the objectives, such that the **selection of contrast types** is made considering **all variables**.

15.5.3. Up to the present section, the contrast types explained have been **intra-measurement** studies; that is, given a **single measurement**, studying the relationships among different variables **within the same measurement** to obtain empirical results to be contrasted with critical values, in order to determine whether an

observed tendency is sufficient in **sufficiently representative samples**, which, if so, would validate the hypothesis for whose acceptance the study was designed—thus becoming the **rational hypothesis, provisionally**, but only within the **accepted error margins** in intra-measurement studies.

15.5.4. From the following section onward, **inter-measurement** studies will begin, i.e., between different measurements—either **inter-measurement intra-individual** (different measurements of the **same** subject or option, Sections **16** and **17**) or **inter-measurement inter-individual** (especially in Section **19**), considering different measurements of **different** subjects or options that will be related to each other.

15.5.5. Up to now, for intra-measurement studies we have been combining the exposition of critical contrasts both for the most normal cases—those whose dispersion ranges between zero and the maximum—as well as for the more exceptional **omega** cases, where in an **N** greater than two there is a number of ideals between two and **N – 1**, which will require additional critical tests insofar as these are empirical models whose **ideal dispersion** will tend to differ from the maximum as the omega magnitude of the ideals increases.

15.5.6. From the following Section Sixteen onward—where inter-measurement studies begin with the study of **necessary, possible**, and **real** variations; in Section Seventeen, where **linear predictions** (ideal or empirical) are studied; and in Section Nineteen, where inter-measurement studies are carried out, both **intra-individual** and **inter-individual**—those sections will address inter-measurement studies for models whose dispersion ranges between zero and the maximum, whether for the study of equality or of bias (positive or negative). In Section Twenty, tests will be developed for **inter-measurement rational critique**, both intra-individual and inter-individual, **under omega conditions**. The models in Section Eighteen on **ranking studies**, insofar as they include **tied nth positions**, will in themselves facilitate study in models that include a magnitude of ideals between two and **N – 1**.

15.5.7. The main difference between studies under the **maximum-dispersion assumption** and studies under **omega conditions** is that the former start from *a priori*

knowledge of the **maximum tendencies** of a model, whereas studies that contemplate a number of ideals between two and **N – 1** can only determine the **ideals a priori**.

15.5.8. While in proportional studies under the maximum-dispersion assumption every **individual or sample proportion** equals the quotient of the **real empirical value** over the **maximum theoretical or empirical tendency** (so the quotient is a proportion that, in an **error** study, must be **equal to or less than** the error margin, or in a **reliability** study **equal to or greater than** the reliability margin—with the adaptation that, to convert error proportions into reliability, the **error proportion is subtracted from one**—the maximum possible proportion between two factors in a quotient—and the differential is contrasted against a reliability margin), in **omega** studies there are in fact **no maximum values**, since only the **ideal values** are known. Therefore, in **omega proportions**, instead of computing the **proportion of real to ideal**, what must be computed is the **similarity between real and ideal**; the greater the identity of real and ideal within an error margin, the more the real tendency is accepted as sufficiently ideal.

15.5.9. For Sections Sixteen and Seventeen, the most relevant concepts are the **variations**—necessary, possible, and real—where, **under omega conditions**, the **necessary variation** between an **ideal** subject or option and the **ideal value** should equal the **absolute value of the difference** between its empirical probability and the **inversion of omega**. Based on the necessary variation, the **Possible Variation by Chance** is the necessary variation times an **empirical or theoretical error margin**; and the **Possible Reliable Variation** equals the necessary variation times an **empirical or theoretical reliability margin**. On the basis of the Possible Variation—by Chance or Reliable—one determines whether the **real variation observed** between two measurements for an **ideal** subject or option lies within what would be by chance or reliable, according to the explanations detailed below, which for omega models will be expanded and adapted in Section Twenty.

16. Rational Critique of Possible Variation

16.1. Necessary Variation, Possible Variation (by Chance or Reliable), Critical or Empirical, Individual; and Real Variation

16.1.1. Necessary Variation

16.1.1.1. Necessary variation is that which occurs between a subject's or option's **empirical probability** and any other **individual statistic** or the **empirical probability** of another subject or option. It is understood that the **Bias Level**—relative or normal—between a subject's empirical probability and any other individual statistic (including the **inversion of N**) or any other empirical probability is **equal to the necessary variability** between both terms. It follows that any Bias Level—normal or relative—between two empirical probabilities, or between an empirical probability and an individual statistic, measures—besides bias—the **necessary variation** between both factors, **provided** the Bias Level is interpreted in **absolute value** or the factors are ordered so that it is **greater minus lesser**. It is “necessary” in the sense that the **necessary variation added to the lesser** equals the greater, or **subtracted from the greater** equals the lesser.

16.1.1.2. The Bias Level, in itself, has **two functions**:

16.1.1.2.1. First, it measures **bias**.

16.1.1.2.2. Second, if interpreted in **absolute value** or with factors ordered as **greater minus lesser**, it measures the **necessary variation** between the factors for which the Bias Level is estimated.

16.1.1.3. The Bias Level, as **necessary variability**, measures the **necessary variation** between the Bias Level's factors.

16.1.1.3.1. As necessary variability between **two factors**, the (absolute) value of the Bias Level—normal or relative—**added to the lesser** of the factors from which the Bias Level was computed equals the **greater** of those factors:

$| p(x+) - p(x-) | = \text{necessary variation between the maximum and the minimum}$

$| p(x+) - p(x-) | + p(x-) = p(x+)$

16.1.1.3.2. As necessary variation between two factors, the (absolute) value of the Bias Level **subtracted from the greater** of the factors equals the **lesser** of the factors:

$| p(x+) - p(x-) | = \text{necessary variation between the maximum and the minimum}$

$p(x+) - | p(x+) - p(x-) | = p(x-)$

16.1.1.3.3.

Synthesis

16.1.1.3.3.1. The Bias Level, as necessary variation, measures the **necessary variability** between two factors: **adding** the absolute necessary variability to the lesser factor yields the greater; **subtracting** it from the greater yields the lesser.

16.1.1.3.3.2. The Bias Level, as necessary variability, measures the **distance** of each factor from the other factor; that distance is the **necessary variation**.

16.1.1.3.3.3. The necessary variation is the variability that would be required for the **greater** factor to **decrease** to equal the lesser, and the variability that would be required for the **lesser** factor to **increase** to equal the greater.

16.1.1.3.3.4. Thus, with respect to the greater factor (in terms of **decrease** in its probability) and with respect to the lesser factor (in terms of **increase** in its probability), the necessary variation is that variability required for each factor to become **equal to the other**.

16.1.1.3.3.5. The necessary variation, with respect to the **greater** factor, measures how much the greater's empirical probability would need to **decrease** to equal the lesser.

16.1.1.3.3.6. The necessary variation, with respect to the **lesser** factor, measures how much the lesser's empirical probability would need to **increase** to equal the greater.

16.1.1.3.3.7. The necessary variation of the **greater**, relative to the lesser, thus measures the **necessary decrease** in the greater's probability to equal the lesser.

16.1.1.3.3.8. The necessary variation of the **lesser**, relative to the greater, thus measures the **necessary increase** in the lesser's probability to equal the greater.

16.1.1.4. Necessary variation between an **empirical probability** and the **inversion of N** equals the **absolute value of the normal Bias Level**:

$$| p(x) - 1/N |$$

16.1.1.5. Necessary variation between the **Maximum Possible Empirical Probability** and an empirical probability equals **one minus** that empirical probability:

$$(1 - p(x_i))$$

16.1.1.6. Necessary variation between an **empirical probability** and the **Minimum Possible Empirical Probability** equals the empirical probability itself:

$$(p(x_i) - 0) = p(x_i)$$

16.1.1.6.1. The necessary variation will **always** equal the **absolute value of the Bias Level**—normal or relative—of the factor under study; thus, in reality, every absolute Bias Level is the **necessary variation** for the greater to **decrease** enough to equal the lesser, or for the lesser to **increase** enough to equal the greater.

16.1.1.7. However, in reality, the fact that something is **necessary** does not imply that it is **absolutely possible**; indeed, often, in reality, what is necessary may even be **impossible** within an **insufficient or non-infinite** time. More commonly, given conditions of **possibility**—theoretical or empirical—the necessary will be at least **partly possible**, though only partly so in theory or empirically. Since statistics works

with **samples**, it works only with **margins of possibility**; only in the **infinite** is even the impossible inevitable (theoretically). But in statistics, human limitation makes it impossible to work directly with the infinite universe; statistics studies a **partial reality**—the **sample**—where not everything possible or impossible is always inevitable, since a sample, by definition, is **not infinite**. Every sample of knowledge is knowledge **limited to the sample**, within the **possible margins** of the sample. In a partial reality there are only margins of possibility; in samples one must study only the **possible margins of possibility**.

16.1.1.8. In **Possible Variation**, we will study, in the sample, that portion of the **necessary variation** that is, **critically or empirically**, either a necessary variation **partly possible by chance**, or **reliably**, at the **individual** or **sample** level—beginning with the expositions on **Possible Variation—by Chance or Reliable, Critical or Empirical, Individual**.

16.1.2. The Possible Variation

16.1.2.1. A necessary variation—a Bias Level, normal or relative—or the necessary variation between the **Maximum** or **Minimum Possible Empirical Probability** and a given empirical probability measures the **necessary variation** by which the **greater** would have to **decrease** to equal the **lesser** (or the Minimum Possible Empirical Probability), or the **necessary variation** by which the **lesser** would have to **increase** to equal the **greater** (or the Maximum Possible Empirical Probability).

16.1.2.2. The **necessary variation** is the variation that would have to occur for one term to become equal to another—either the **necessary decrease** from the greater to the lesser, or the **necessary increase** from the lesser to the greater.

16.1.2.3. But what is necessary, within a limited partial reality, is not always actually possible; what is **actually possible** will be that part of the necessary that can occur in the partial reality. What must be assessed is whether a **possible variation** is sufficient to be **reliable**, or whether it is a variation produced by **chance**. Nothing is immutable: reality is the eternal becoming of movement in real movement; the idea of movement is dialectic itself. Dialectically, reality is formed by infinite variations in magnitude—of intensity or occurrence—of what happens (sensibly, events or phenomena), where

there are variations that can be **random** (chance itself as a cause of magnitude variations) and variations that, while possibly also the product of chance, can nonetheless amount to **sufficient reason** to be truly **reliable** variations; although, it is true, there may also be variations that, without being defined by chance, are still not sufficiently reliable.

16.1.2.4. A **necessary variation** only measures **what would be necessary**.

16.1.2.5. Not everything necessary is always possible. Given **real conditions of possibility**, these conditions may be **critical** or **empirical**. **Critical conditions** of possibility are determined by **scientific policy** according to its political definition of the possible, established in **critical reason**; **empirical conditions** of possibility are determined by the **rate of empirical dispersion**.

16.1.2.6. Everything necessary will be possible **provided** there are theoretical or empirical conditions of possibility that allow the necessary to occur. The question is whether the **possible part** of the necessary that occurs is **sufficiently reliable**, or merely a **random** variation that does not constitute real reliability. Only that event will be **reliably conditioned** which—being of random (natural) origin or of scientific experimental origin—is capable of producing a **reliable regularity**.

16.1.2.7. If the theoretical or empirical conditions of possibility that would allow the necessary event to occur **do not hold**, then the necessary event **will not be possible**. Throughout history one observes millions of events that would have been necessary—theoretically or empirically—but were not actually possible, or only partly possible. Often only a **percentage** of the necessary can be achieved; from the 100% of the necessary, only a **part** can be realised. The necessary will be **wholly or partially possible** given suitable conditions of possibility. In statistical probability, it is normally difficult to have conditions that guarantee **100%** of the necessary. Most commonly in *Impossible Probability*, within the **sample**, the necessary is only **partially possible**, to the extent that theoretical or empirical conditions allow at least **a part** of the necessary to be possible.

16.1.2.8. In limited human reality there are **levels of possibility**; within sample reality there are **levels of inevitability**. The necessary is not always 100% possible; often only

a part of the necessary can be realised. Real (theoretical or empirical) conditions of possibility determine the **proportion of fulfilment** of the necessary.

16.1.2.9. Something may be necessary and be **only partially possible**, critically or empirically.

16.1.2.10. Not everything necessary must be absolutely possible, but it **can** be possible in **some part**, level, degree, proportion, or rate of what is necessary. That is, within **critical or empirical** conditions of possibility, perhaps not all the necessary is possible—inasmuch as real (empirical or theoretical) conditions do not allow it—but at least **a part** of the necessary may be possible.

16.1.2.11. Given a **necessary variation** between two factors, only that **possible variation** of truly **critical or empirical** nature will be accepted in reality. There are **two types** of possible variation according to the distinction between **theory** and **empirical reality**:

- **Critical possible variation**, established by the **political theory of science**; and
- **Empirical possible variation**, established from the **rate of empirical dispersion**.

Each model (critical or empirical) can in turn be subdivided into **two categories**, depending on whether they describe a **real variation** that is merely a **product of chance** (which need not imply any regular pattern of reliability)—i.e., **chance itself**—or a **real variation** which, although of random natural origin or experimental scientific origin, **becomes a reliable variation** (critical or empirical), i.e., **sufficiently reliable** in theory or in fact.

16.1.2.12. **Critical possible variation** is that part of the **necessary variation** defined according to the **ideals of the political theory of science**—i.e., necessary variability that **should** be possible in order to be truly **critical**. The **critically possible necessary variation** will define when a variation is **by chance**, according to the **Individual Critical Possible Variation by Chance**, intended to critically study random variations in matter produced by chance. The **Individual Critical Possible Variation by Chance** equals the **necessary variation multiplied by the error percentage X divided by one hundred**.

The **Critical Reliable Possible Variation** is the model of variation which—though possibly caused randomly by a natural combination of accidents or contingencies, or caused by experimental scientific manipulation—is capable of being **sufficiently reliable**. The **Critical Reliable Possible Variation** equals the **necessary variation multiplied by the reliability percentage X divided by one hundred**.

16.1.2.13. Empirical possible variation is that part of the **necessary variation** that is possible **as a matter of fact**. Two models are likewise distinguished: depending on whether its nature **does not** follow any pattern of reliable regularity (hence **by chance**), or whether—though it may be originated by random natural accidental contingencies or experimental scientific variables—it is a **reliable variation**. Thus we distinguish between:

- **Individual Empirical Possible Variation by Chance** = **necessary variation × empirical error**, to study variations due to **empirical chance**; and
- **Empirical Reliable Possible Variation** = **necessary variation × empirical reliability**, to study whether a phenomenon is truly **reliable**.

16.1.2.14. Possible Variation by Chance may be **theoretical** (i.e., critical) or **empirical**, and measures whether a variation is a **random** product of reality **by mere chance**, which need not imply a pattern of regularity.

16.1.2.15. Reliable Possible Variation may likewise be theoretical (critical) or empirical, and measures whether a given variation—product of movement, random, chaotic, natural (chance in matter), or experimental scientific—has entailed a sufficiently reliable transformation of behaviour of history and within history.

16.1.2.15.1. In experimental studies, where the behaviour of matter does not obey natural chains of accidental contingency and instead responds to manipulation of the independent variable, every change in the behaviour of the dependent variable will be a change produced by experimental manipulation of reality. In experimental studies, what remains is to determine whether a behaviour to be characterised is a random variable or sufficiently reliable.

16.1.2.15.2. A **random variable** is a variable behaviour of matter that follows a **random model** of behaviour implying **no pattern of regularity** other than chance itself. Chance behaviour is **unpredictable** insofar as one can only predict that **anything may happen** under **free will in equality of opportunities**; beyond the mathematical definition, we do not know what may happen in the chance of what occurs in **space-time**.

16.1.2.15.3. A variable is **reliable** when **what may happen is predictable**, even if its origin was random or the product of a manipulation of reality—logically accepting an **error margin** within which everything is possible.

16.1.2.16. The structure—theoretical or empirical—of chance and reliability

16.1.2.16.1. The theoretical structure of chance and reliability

16.1.2.16.1.1. Scientific policy defines variables according to its **political theory of science**, upon which it establishes **critical reason**; the **ideal critical reason** is the **critical probability**, defined on the foundations of the political/ideological theory of scientific policy.

16.1.2.16.1.2. A **critical probability** will be an **error** or **reliability** margin depending on whether the percentage $X/100$ is a **percentage of error** or **of reliability**.

16.1.2.16.1.3. A **percentage X of error**, divided by one hundred, times a **maximum tendency** equals the **error margin** that scientific policy is prepared to accept for that maximum tendency. If the **ideal** is **unity** itself (e.g., in **Critical Bias Level** or in **Critical Proportions**), then the **critical probability** over the **unit** of the quotient (the maximum tendency of the quotient) is **$X : 100$** .

16.1.2.16.1.4. A **percentage X of reliability**, divided by one hundred, times an **ideal tendency** equals the **reliability margin** that scientific policy is prepared to accept over that ideal tendency. If the ideal tendency is the **unit of the quotient** itself (e.g., in **Critical Bias Level** or in **Critical Proportions**), then **$X : 100$** is itself the **reliability margin**.

16.1.2.16.1.5. If scientific policy defines **error** or **reliability** in terms of **X%**—set ideologically within its political theory of science—to define variables of random reality and decide whether a variation within the accidental movement of matter is **by chance**

or a **sufficiently reliable accident**, then, given a **necessary variation**, the **necessary variation itself** is the **maximum or ideal tendency** possible.

16.1.2.16.1.6. The product of the **necessary variation** by the **percentage X of error / 100** equals the **Individual Critical Possible Variation by Chance**; real variations in objective reality should be contrasted against this to decide whether a change in a behavioural pattern of matter is or is not **by chance**, according to the **real change in magnitude**—i.e., a behavioural change is **by chance** if the **real variation** is **equal to or less than** the **Individual Critical Possible Variation by Chance**.

16.1.2.16.1.7. The product of the **necessary variation** by the **percentage X of reliability / 100** equals the **Critical Reliable Possible Variation**, to contrast **objective real variations** and decide whether a **real change** in magnitude shows behaviour in reality—accidental or experimental—sufficient to be **reliable, significant, rational**, whenever the **real change** is **equal to or greater than** the **Critical Reliable Possible Variation**.

16.1.2.16.2. The empirical structure of chance and reliability

16.1.2.16.2.1. Theoretically, a **theoretical margin** (critical reason) will be an **error or reliability** margin depending on whether the **percentage X/100** is a percentage of **error** or of **reliability**. If **X/100** is an **error** percentage, it is an **error margin**; if **X/100** is a **reliability** percentage, it is a **reliability margin**.

16.1.2.16.2.2. At the **Critical Sample Level of Equality**, a **percentage X of error / 100** times the **Maximum Theoretical Possible Deviation** (Mean or Standard) equals the **error margin** within which it is accepted that, **if a Mean or Standard Deviation (DMoS) is equal to or less than the critical reason**, the model is one of **equality of opportunities**.

16.1.2.16.2.3. At the **Critical Sample Level of Bias**, a **percentage X of reliability / 100** times the **Maximum Theoretical Possible Deviation** (Mean or Standard) equals the **reliability margin** within which it is accepted that, **if a Mean or Standard Deviation (DMoS) is equal to or greater than the critical reason**, the model is one of **bias**.

16.1.2.16.2.4. Thus, if at the **Critical Sample Level of Equality** the Mean or Standard Deviation (DMoS) must be **equal to or less than** an **error margin**, and at the **Critical Sample Level of Bias** the Mean or Standard Deviation (DMoS) must be **equal to or greater than** a **reliability margin**, then under equality of opportunities the Mean or Standard Deviation is itself an **empirical error margin** that must be $\leq$ the **theoretical error margin**, whereas in the study of bias the Mean or Standard Deviation is itself an **empirical reliability margin** that must be $\geq$ a **theoretical reliability margin**.

16.1.2.16.2.5. Insofar as, in normal models, dispersion is a variable that ranges **between zero and the maximum**, and according to the object of study, dispersion is interpreted differently depending on whether it ranges toward **zero** or toward the **maximum**.

16.1.2.16.2.6. Every equality-of-opportunities model whose ideal is **zero dispersion** will regard dispersion as a factor of **error**, whereas those whose object is **maximum possible dispersion** (when, out of N, there is only one subject or option to be raised to maximum empirical probability) will regard dispersion itself as a factor of **reliability**.

16.1.2.16.2.7. Therefore, in normal models—where dispersion ranges between **zero** and **maximum**—if the ideal is **zero**, dispersion is in itself a factor of **error**; and if it should tend to the **maximum**, dispersion is in itself a factor of **reliability**.

16.1.2.16.2.8. For any equality-of-opportunities model, insofar as dispersion is interpreted as an **error factor in itself**, the Mean or Standard Deviation (DMoS) is the **empirical sample error factor**. The empirical sample **error rate** for these studies will be called the **First Modality of Empirical Error**, and denoted by the first Greek letter **alpha, α** : the quotient of **Mean or Standard Deviation, DMoS** (as an empirical error factor in itself) over **Maximum Theoretical Possible Mean Deviation** or **Maximum Theoretical Possible Standard Deviation**, depending on whether the numerator is Mean or Standard Deviation.

α = First Modality of Empirical Error

16.1.2.16.2.8.1. First Modality of Empirical Error, α , with Mean Deviation (**DM**), in equality of opportunities:

$$\alpha = DM : \{ [(1 - 1/N) \cdot 2] : N \}$$

DM= Mean Deviation

16.1.2.16.2.8.2. First Modality of Empirical Error, α , with Standard Deviation **S**, in equality of opportunities:

$$\alpha = S : \sqrt{[(1 - 1/N)^2 + ((1/N)^2 \cdot (N - 1))]} : N \}$$

S= Standard Deviation

16.1.2.16.2.9. Thus, if the object is equality of opportunities, the First Modality of Empirical Error is **alpha**: since the ideal is to tend to **zero** error (dispersion), the **error rate**—the quotient of **sample empirical dispersion** over the **maximum possible sample dispersion**—should tend to **zero**. **Reliability** will then equal **one minus the error rate (dispersion)**; hence the **First Modality of Empirical Reliability, not-alpha, $\neg\alpha$** (unit minus alpha), i.e., **one minus the quotient of Mean or Standard Deviation over Maximum Theoretical Possible Mean or Standard Deviation**.

$\neg\alpha = 1 - \alpha = \text{First Modality of Empirical Reliability}$

16.1.2.16.2.9.1. First Modality of Empirical Reliability, $\neg\alpha$, with Mean Deviation, **DM**, in equality study:

$$\neg\alpha = 1 - \{ DM : \{ [(1 - 1/N) \cdot 2] : N \} \}$$

DM= Mean Deviation

16.1.2.16.2.9.2. First Modality of Empirical Reliability, $\neg\alpha$, with Standard Deviation, **S**, in equality study:

$$\neg\alpha = 1 - \{ S : \sqrt{ \{ [(1 - 1/N)^2 + ((1/N)^2 \cdot (N - 1))] : N \} } \}$$

Standard Deviation

16.1.2.16.2.10. And if **alpha** is sample empirical dispersion over maximum sample empirical dispersion—inasmuch as dispersion is itself an **error factor** to guarantee equality—then, if the object of study were the opposite, i.e., to **increase dispersion to the maximum possible**, dispersion will be interpreted positively as **reliability in itself**.

16.1.2.16.2.11. If the object of study is to increase the sample empirical dispersion to the **maximum possible**, in those models where out of N there is **only one** subject or option ideally tending to **Maximum Possible Empirical Probability**, the ideal will be to tend to **Maximum Theoretical Possible Mean Deviation** or **Maximum Theoretical**

Possible Standard Deviation; thus, for these models, dispersion is in itself a **reliability** factor.

16.1.2.16.2.12. If, in studies tending to **maximum dispersion**, dispersion is itself a **reliability factor**, the **Second Modality of Empirical Reliability, beta, β** (second Greek letter), equals **Mean or Standard Deviation (DMoS)** over **Maximum Theoretical Possible Mean or Standard Deviation**.

β = Second Modality of Empirical Reliability

16.1.2.16.2.12.1. Second Modality of Empirical Reliability, **β** , with Mean Deviation, **DM**,
in bias study:

$$\beta = DM : \{ [(1 - 1/N) \cdot 2] : N \}$$

DM= Mean Deviation

16.1.2.16.2.12.2. Second Modality of Empirical Reliability, **β** , with Standard Deviation, **S**,
in bias study:

$$\beta = S : \sqrt{\{ [(1 - 1/N)^2 + ((1/N)^2 \cdot (N - 1))] : N \}}$$

S= Standard Deviation

16.1.2.16.2.13. And if, logically, in models tending to maximum dispersion the **Second Modality of Empirical Reliability** is sample empirical dispersion over the **maximum possible sample dispersion**, then necessarily the **Second Modality of Empirical Error, not-beta, $\neg\beta$** , equals **one minus** the quotient of **Mean or Standard Deviation (DMoS)** over **Maximum Theoretical Possible Mean or Standard Deviation**.

$$\neg\beta = 1 - \beta = \text{Second Modality of Empirical Error}$$

16.1.2.16.2.13.1. Second Modality of Empirical Error, $\neg\beta$, with Mean Deviation, **DM**, in bias study:

$$\neg\beta = 1 - \{ DM : \{ [(1 - 1/N) \cdot 2] : N \} \}$$

DM=Mean Deviation

16.1.2.16.2.13.2. Second Modality of Empirical Error, $\neg\beta$, with Standard Deviation, **S**, in bias study:

$$\neg\beta = 1 - \{ S : \sqrt{ [[(1 - 1/N)^2 + ((1/N)^2 \cdot (N - 1))] : N] } \}$$

S= Standard Deviation

16.1.2.16.2.14. In this way, **universally**, the **critical probability** is the magnitude of the **moral variable X** divided by one hundred; the moral variable X is interpreted as **error** or **reliability** depending on the model or rational-critique test—so the critical probability may be an **error margin** or a **reliability margin** depending on whether X is a percentage of error or of reliability—but universally the **critical probability = X : 100**, regardless of how the moral variable is valued.

16.1.2.16.2.15. The quotient **Mean or Standard Deviation (DMoS) / Maximum Theoretical Possible Mean or Standard Deviation** is the same, identical quotient both for the **First Modality of Empirical Error** and for the **Second Modality of Empirical Reliability**.

16.1.2.16.2.16. The **only difference** between **alpha** and **beta**—the only difference in interpreting the quotient of **sample empirical dispersion** over the **maximum possible**—is that, in **equality of opportunities**, **DM / Max Theoretical Possible DM** or **S / Max Theoretical Possible S** is in itself an **empirical sample error rate** (since the ideal is to tend to **zero** dispersion as an error factor in itself), whereas in models tending to **maximum possible sample dispersion**, **DM** (or **S**) itself is a **reliability margin**; hence **DM / Max Theoretical Possible DM** or **S / Max Theoretical Possible S** is an **empirical reliability rate**.

16.1.2.16.2.17. Since, in **error** studies, the **empirical error rate—First Modality of Empirical Error, alpha**—equals **sample empirical dispersion / maximum possible sample dispersion**, then in equality-of-opportunities studies any **empirical error rate** $\leq X : 100$ would be accepted; equivalently, the **difference from one** (i.e., one minus that rate) would be accepted if $\geq X : 100$ of reliability—this is the **Theoretical Critical Sample Proportion of Equality**.

16.1.2.16.2.18. Likewise, since **beta—Second Modality of Empirical Reliability**—is **sample empirical dispersion / maximum possible sample dispersion**, and is an **empirical reliability rate** in models of **maximum growth of dispersion** (when, out of N, there is only **one** possible ideal subject or option), then whenever the **Second Modality of Empirical Reliability** is $\geq X : 100$ of reliability it would be accepted—i.e., the **Theoretical Critical Sample Proportion of Bias**.

16.1.2.16.2.19. However, given an **empirical error rate** in equality of opportunities, or an **empirical reliability rate** in a bias study—**whether accepted or rejected**—that rate is the **real rate** for that particular measurement, because it is the true rate manifested by that specific sample. Accepted or not, it cannot be denied that this is the **only real rate** in that particular measurement for that sample. Therefore, any **a priori estimation of future behaviour** (prediction) that aims to be realistic about real empirical values must estimate using **that rate and not some other**, because **that** is the sample's true rate. If a sample shows a given **growth or decrease rate**, then to make empirical projections sufficiently realistic about what may happen in the future in that sample—under this behavioural tendency—**empirical projections** must be made **from the true proportions that the sample itself exhibits**.

16.1.2.17. Once it is determined what the **First Modality of Empirical Error** is (Mean or Standard Deviation, as an **error margin** in equality) over **Maximum Theoretical Possible Mean or Standard Deviation**, hence the **First Modality of Empirical Reliability** (unit minus the First Modality of Empirical Error), and what the **Second Modality of Empirical Reliability** is (under the assumption of **maximum dispersion**: Mean or Standard Deviation, as a **reliability margin**, over **Maximum Theoretical Possible Mean or Standard Deviation**), hence the **Second Modality of Empirical Error** (unit minus the Second Modality of Empirical Reliability)—

16.1.2.18. The **Empirical Possible Variation by Chance** equals the product of the **Necessary Variation** by the **First or Second Modality of Empirical Error**, according to the object of study (**equality of opportunities** or **tendency to maximum possible dispersion**).

16.1.2.19. And the **Empirical Reliable Possible Variation** equals the product of the **Necessary Variation** by the **First or Second Modality of Empirical Reliability**, depending on the object of study (**equality of opportunities** or **tendency to maximum possible dispersion**).

16.1.3. The real variation

16.1.3.1. There is said to be **real variation** between two measurements when a real difference is observed between them; any measurement minus any other distinct

measurement, whose **difference** between the two measurements is **non-zero**, is in itself a **real variation**. What remains to be determined is whether that real variation produced between the two measurements is **by chance** or **reliably** real. Depending on the object of study—**equality of opportunities** or **bias** (positive or negative)—the observed real variations must be **contrasted against the Possible Variations**. Within each object of study, and **only within each object of study**, one decides whether a real variation is **by chance** or **reliable** if, under the appropriate rational-critique modality (contrast) for that object of study, the real variation is $\leq$ the **Possible Variation by Chance** (Critical or Empirical), or $\geq$ the **Reliable Possible Variation** (Critical or Empirical), in order to decide whether, for that object of study, that particular real variation is by chance or reliable.

16.1.3.1.1. Real variation in equality of opportunities

16.1.3.1.1.1. Individual level

16.1.3.1.1.1.1. There is real variation toward equality of opportunities, at the individual level, **whenever the absolute value of the Bias Level in the second measurement is less than** the absolute value of the Bias Level in the first measurement.

16.1.3.1.1.1.2. If the **difference between the absolute values of bias**—provided the second measurement's absolute bias is less than the first's—is $\leq$ the **Individual Critical or Empirical Possible Variation by Chance**, it will be said that there has been a **chance change** in behaviour toward equality of opportunities (empirical or theoretical, or both).

16.1.3.1.1.1.3. If the **difference between the absolute values of bias**—again with the second measurement's absolute bias less than the first's—is $\geq$ the **Individual Critical or Empirical Reliable Possible Variation**, it will be said that there has been a **reliable change** in behaviour toward equality of opportunities (critical or empirical, or both).

16.1.3.1.1.2. Sample level

16.1.3.1.1.2.1. There is real variation toward equality of opportunities at the **sample** level whenever the **Mean or Standard Deviation (DMoS)** of the **second** measurement is **less than** that of the **first** measurement.

16.1.3.1.1.2.2. If the real variation toward equality of opportunities—**sample dispersion of the first measurement minus that of the second**—is $\leq$ the **Sample Critical or Empirical Possible Variation by Chance**, then there has been a **chance** change in sample behaviour toward equality of opportunities (critical or empirical, or both).

16.1.3.1.1.2.3. If the real variation toward equality of opportunities—**sample dispersion of the first measurement minus that of the second**—is $\geq$ the **Sample Critical or Empirical Reliable Possible Variation**, then there has been a **reliable** sample-level change in behaviour toward equality of opportunities (critical or empirical, or both).

16.1.3.1.2. Real variation in the study of bias

16.1.3.1.2.1. Real variation of positive bias at the individual level

16.1.3.1.2.1.1. Whenever the **empirical probability in the second measurement is greater** than in the first, there is a real variation **increasing positive bias**.

16.1.3.1.2.1.2. Any real variation increasing positive bias—**empirical probability (second) minus empirical probability (first)**, provided the second is greater—if that difference is $\leq$ the **Individual Critical or Empirical Possible Variation by Chance**, it will be said there has been a **chance** change in individual behaviour toward greater positive bias (empirical or critical, or both).

16.1.3.1.2.1.3. Any real variation increasing positive bias—**empirical probability (second) minus (first)**, provided the second is greater—if that difference is $\geq$ the **Individual Critical or Empirical Reliable Possible Variation**, it will be said there has been a **reliable** change in individual behaviour toward greater positive bias (critical or empirical, or both).

16.1.3.1.2.2. Real variation of negative bias at the individual level

16.1.3.1.2.2.1. Whenever the **empirical probability of the second measurement is less** than that of the first, there is a **real individual variation increasing negative bias**.

16.1.3.1.2.2.2. If the **difference of empirical probability (first) minus (second)**—with the first greater—is $\leq$ the **Individual Critical or Empirical Possible Variation by Chance**, it will be said there has been a **chance** change in individual behaviour **increasing negative bias** (critical or empirical, or both).

16.1.3.1.2.2.3. If the **difference of empirical probability (first) minus (second)**—with the first greater—is $\geq$ the **Individual Critical or Empirical Reliable Possible Variation**, it will be said there has been a **reliable individual increase of negative bias** (critical or empirical, or both).

16.1.3.1.2.3. Real variation of bias at the sample level

16.1.3.1.2.3.1. At the sample level there is **real variation of bias** if the **Mean or Standard Deviation (DMoS)** of the **second** measurement is **greater** than that of the **first** measurement.

16.1.3.1.2.3.2. If the **difference of sample empirical dispersion—second measurement (being greater) minus the first**—is $\leq$ the **Sample Critical or Empirical Possible Variation by Chance**, there has been a **chance** change in sample behaviour **in favour of bias** (critical or empirical, or both).

16.1.3.1.2.3.3. If that **difference—second measurement (being greater) minus the first**—is $\geq$ the **Sample Empirical or Critical Reliable Possible Variation**, there has been a **reliable** sample-level change in behaviour **in favour of bias** (critical or empirical, or both).

16.1.3.2. It may occur that a behavioural change—by chance or reliable, individual or sample-level—is **critical but not empirical**, or **empirical but not critical**; ideally, a change by chance should be **both critical and empirical**, and a reliable change should be **reliable both critically and empirically**.

16.1.3.3. Ideally, the **real variation** would be **reliable both critically and empirically**, at both **individual** and **sample** levels, although there are events that, while **not reliable**, are **not random** either.

16.1.3.4. As a preliminary clarification: if an equation is said to be **valid only for equality of opportunities**, it is valid **only** for equality of opportunities and **not** for any

other object of study. If an equation is said to be valid **only** for the study of positive bias, it applies **only** to positive bias, and **not** to other objects of study. If an equation is said to be valid **only** for the study of negative bias, it applies **only** to negative bias, and **not** to other objects of study. **According to the object of study, the rational-critique model must be selected**, and one may not select a rational-critique model that does **not match** the object of study.

16.1.3.5. In particular, for **Individual** contrasts of **Possible Variation by Chance** (Critical or Empirical) for studies of **equality** or **bias** (positive or negative): apply the **Possible Variation by Chance** (Critical or Empirical) for the corresponding object of study **only after** demonstrating that: in **equality** studies the **absolute Bias Level in the second measurement is less than** in the first; in **positive bias** studies the **empirical probability in the second measurement is greater than** in the first; in **negative bias** studies the **empirical probability in the second measurement is less than** in the first.

16.1.3.6. If, in equality of opportunities, the second measurement **does not** have an **absolute bias value less than** the first, **do not apply** the **Individual Possible Variation by Chance** (Critical or Empirical) for equality.

16.1.3.7. If, in the study of positive bias, the second measurement's empirical probability **is not greater** than the first's, **do not apply** the **Individual Possible Variation by Chance** (Critical or Empirical) for positive bias.

16.1.3.8. If, in the study of negative bias, there is **no real variation increasing negative bias** in the second measurement relative to the first, then **do not apply** the **Individual Possible Variation by Chance** (Critical or Empirical) for negative bias.

16.2. Individual Critical Possible Variation contrasts, in measurements with equal N

16.2.1. Contrast of Individual Critical Possible Variation by Chance — legend:

$p(xi/1^a)$ = empirical probability of the subject or option in the **first** measurement

$p(xi/2^a)$ = empirical probability of the subject or option in the **second** measurement

16.2.1.1. Contrast of Individual Critical Possible Variation by Chance in equality of opportunities.

Provided the **absolute value of bias** in the **second** measurement is **less than** in the **first**:

$$\left(|p(xi/1^a) - 1/N| \cdot p(xc) \right) - \left(|p(xi/1^a) - 1/N| - |p(xi/2^a) - 1/N| \right) = \text{zero or positive} \rightarrow \text{accepted by chance}$$

$$p(xc) = X : 100$$

X = error percentage

16.2.1.2. Contrast of Individual Critical Possible Variation by Chance in the study of positive bias.

Provided the empirical probability in the second measurement is **greater** than the first:

$$\left((1 - p(xi/1^a)) \cdot p(xc) \right) - \left(p(xi/2^a) - p(xi/1^a) \right) = \text{zero or positive} \rightarrow \text{increase of positive bias accepted by chance}$$

$$p(xc) = X : 100$$

X = error percentage

16.2.1.3. Contrast of Individual Critical Possible Variation by Chance, in the study of negative bias.

Provided the empirical probability in the **first** measurement is **greater** than in the **second**:

$$\left[(p(xi/1st) - 0) \cdot p(xc) \right] - (p(xi/1st) - p(xi/2nd)) = \text{zero or positive} \rightarrow \text{increase of negative bias accepted by chance}$$

$$p(xc) = X : 100$$

X = error percentage

16.2.2. Individual Critical Reliable Possible Variation

16.2.2.1. Contrast of Individual Critical Reliable Possible Variation, in the study of equality of opportunities:

$[| p(x_i/1st) - 1/N | - | p(x_i/2nd) - 1/N |] - [| p(x_i/1st) - 1/N | \cdot p(x_c)] = \text{zero or positive} \rightarrow$
equality of opportunities accepted

$$p(x_c) = X : 100$$

X = reliability percentage

16.2.2.2. Contrast of Individual Critical Reliable Possible Variation, in the study of positive bias:

$(p(x_i/2nd) - p(x_i/1st)) - [(1 - p(x_i/1st)) \cdot p(x_c)] = \text{zero or positive} \rightarrow$ reliable increase of
positive bias accepted

$$p(x_c) = X : 100$$

X = reliability percentage

16.2.2.3. Contrast of Individual Critical Reliable Possible Variation, in the study of negative bias:

$(p(x_i/1st) - p(x_i/2nd)) - [(p(x_i/1st) - 0) \cdot p(x_c)] = \text{zero or positive} \rightarrow \text{reliable increase of negative bias accepted}$

$p(x_c) = X : 100$

X = reliability percentage

16.3. Empirical Possible Variation at the individual level, in measurements with equal N

16.3.1. Individual Empirical Possible Variation by Chance

16.3.1.1. Individual Empirical Possible Variation by Chance via the **First Modality of Empirical Error** (only and exclusively if the object of study is **equality of opportunities**, and **provided** the absolute bias in the second measurement is less than in the first):

$[|p(x_i/1st) - 1/N| \cdot \alpha] - [|p(x_i/1st) - 1/N| - |p(x_i/2nd) - 1/N|] = \text{zero or positive} \rightarrow \text{accepted by chance}$

$\alpha = DM : \{[(1 - 1/N) \cdot 2] : N\}$

DM=Mean Deviation First Measurement

16.3.1.2. Individual Empirical Possible Variation by Chance via the **Second Modality of Empirical Error** (only and exclusively if the object of study is the **tendency to maximum empirical dispersion**).

16.3.1.2.1. Contrast of Individual Empirical Possible Variation by Chance in the study of **positive bias**, via the **Second Modality of Empirical Error**, **provided** the empirical probability of the second measurement is greater than that of the first:

$[(1 - p(x_i/1st)) \cdot \neg\beta] - (p(x_i/2nd) - p(x_i/1st)) = \text{zero or positive} \rightarrow \text{increase of positive bias accepted by chance}$

$$\neg\beta = 1 - \{ DM : \{ [(1 - 1/N) \cdot 2] : N \} \}$$

DM=Mean Deviation First Measurement

16.3.1.2.2. Contrast of Individual Empirical Possible Variation by Chance in the study of **negative bias**, via the **Second Modality of Empirical Error**, **only if** the empirical probability of the first measurement is greater than that of the second:

$[(p(x_i/1st) - 0) \cdot \neg\beta] - (p(x_i/1st) - p(x_i/2nd)) = \text{zero or positive} \rightarrow \text{increase of negative bias accepted by chance}$

$$\neg\beta = 1 - \{ DM : \{ [(1 - 1/N) \cdot 2] : N \} \}$$

DM=Mean Deviation First Measurement

16.3.2. Individual Empirical Reliable Possible Variation

16.3.2.1. Individual Empirical Reliable Possible Variation via the **First Modality of Empirical Reliability**, in **equality of opportunities**:

$[| p(x_i/1st) - 1/N | - | p(x_i/2nd) - 1/N |] - [| p(x_i/1st) - 1/N | \cdot \neg\alpha] = \text{zero or positive} \rightarrow$
reliable reduction of bias accepted

$$\neg\alpha = 1 - \{ DM : \{ [(1 - 1/N) \cdot 2] : N \} \}$$

DM=Mean Deviation First Measurement

16.3.2.2. Individual Empirical Reliable Possible Variation via the **Second Modality of Empirical Reliability** (only and exclusively if the object of study is the **tendency to maximum dispersion**).

16.3.2.2.1. Contrast of Individual Empirical Reliable Possible Variation in the study of **positive bias**, via the **Second Modality of Empirical Reliability**:

$(p(x_i/2nd) - p(x_i/1st)) - [(1 - p(x_i/1st)) \cdot \beta] = \text{zero or positive} \rightarrow$ reliable increase of positive bias accepted

$$\beta = DM : \{ [(1 - 1/N) \cdot 2] : N \}$$

DM=Mean Deviation First Measurement

16.3.2.2.2. Contrast of Individual Empirical Reliable Possible Variation in the study of **negative bias**, via the **Second Modality of Empirical Reliability**:

$(p(x_i/1st) - p(x_i/2nd)) - [(p(x_i/1st) - 0) \cdot \beta] = \text{zero or positive} \rightarrow \text{reliable increase of negative bias accepted}$

$$\beta = DM : \{ [(1 - 1/N) \cdot 2] : N \}$$

DM=Mean Deviation First Measurement

16.4. The effect of N

16.4.1. The effect of N on empirical probability

16.4.1.1. The fact that, as **N** increases, **empirical probability tends to zero** demonstrates that, if the **effect of N** on empirical probability is not taken into account, empirical probabilities from samples with **different N** are **not comparable**—whether in universes of **infinitely many subjects/options** or, in universes of **limited options**, when the number of options changes between measurements (e.g., in an election where the **second round** has **fewer candidates** than the first). As soon as the magnitude **N** changes—whether in infinite or finite universes—**empirical probabilities from different measurements with different N cannot be directly compared**; the **effect of N** must be applied.

16.4.1.2. If the effect of N on empirical probability is **not** taken into account, only two kinds of **inter-individual** comparisons can be made:

16.4.1.2.1. Inter-individual comparisons **within the same measurement** (i.e., intra-measurement inter-individual studies).

16.4.1.2.2. Inter-individual comparisons **between different measurements that have the same N** (i.e., inter-measurement inter-individual studies where each measurement has the **same number** of subjects/options—the type of inter-measurement studies

conducted in this section so far). **Intra-individual comparisons** (between two different measurements of the **same** subject/option) must **always** be **inter-measurement**, since only **one** measurement per subject/option is possible in each m-th measurement; thus every intra-individual comparison is necessarily inter-measurement—the question is **whether** the effect of N must be taken into account.

16.4.1.3. Not taking into account the effect of N **limits** the inter-measurement comparison options, whether **inter-individual** or **intra-individual**.

16.4.1.4. The fact that **without** the effect of N only inter-measurement comparisons with **equal N** (inter-individual or intra-individual) are possible **prevents** making inter-measurement comparisons (intra- or inter-individual) between different measurements that **differ in N**.

16.4.1.5. The **only** way to solve this problem—so that comparisons can be made between empirical probabilities of subjects/options belonging to measurements with **different N**—is to **incorporate into the measurement the effect produced by N** on empirical probability.

16.4.1.6. The effect produced by **N** on empirical probability is **inversely proportional** to the magnitude **N**.

16.4.1.6.1. As **N** increases, **empirical probability tends to zero**.

16.4.1.6.2. As **N** decreases, **empirical probability tends to be farther from zero**.

16.4.1.7. The inverse proportional relationship of **N** on empirical probability is **inverse to the N of the measurement** in which the empirical probability was calculated.

16.4.1.8. If an empirical probability **p(xi)** was calculated from the value **N_{mi}** (meaning the **N** of measurement **mi**, denoted simply **N_{mi}**, i.e., the number of subjects/options forming the sample of measurement **mi**), then the empirical probability was obtained within the **theoretical probability** (and theoretical dispersion probability, error probability) **equal to the inversion of N of measurement mi**, i.e., **1/N_{mi}**. Thus the tendency of the empirical probability toward zero is proportional to the tendency of **1/N_{mi}** toward zero. The empirical probability obtained from **N_{mi}** will be denoted **p(xi/mi)**, meaning: empirical probability **p(xi)** obtained in measurement **mi**.

N_{mi} = total number of subjects/options N in measurement **mi**

$1/N_{mi}$ = theoretical probability of N in measurement **mi**

$p(x_i/m_i)$ = empirical probability in measurement **mi**

16.4.1.9. If the empirical probability has been calculated over the **theoretical dispersion margin**—theoretical probability, error probability—produced by the inversion of “ N_{mi} ”, equal to “ $1/N_{mi}$ ”, then this empirical probability will be **conditioned** by this value “ N_{mi} ”.

16.4.1.10. In an intra-individual or inter-individual, inter-measurement study where empirical probabilities with different **N** are related—where one of them is the empirical probability from measurement “ N_{mi} ”, denoted “ $p(x_i/m_i)$ ”—this empirical probability from measurement “ N_{mi} ” will be related to the empirical probability obtained from any measurement “ N_m ”. The symbol “ N_m ” refers to the value of **N** (subjects/options) of the arbitrary measurement “ m ”. The empirical probability of that other arbitrary measurement “ N_m ” will be denoted “ $p(x/m)$ ”, indicating any empirical probability “ $p(x)$ ” from measurement “ m ”. The theoretical probability of measurement “ m ” with number of subjects/options equal to “ N_m ”, different from **N** of measurement “ mi ”, will be the theoretical probability equal to the inversion of “ N_m ”, i.e., “ $1/N_m$ ”.

$N_m \neq N_{mi}$

N_m = total number of subjects or options N in measurement “ m ”

$1/N_m$ = theoretical probability of N in measurement “ m ”

$p(x/m)$ = empirical probability in measurement “ m ”

16.4.1.11. If one wishes to compare the empirical probability obtained from “ N_{mi} ”, i.e., “ $p(x_i/m_i)$ ”, with any other measurement—say some measurement “ N_m ”, where “ N_m ” indicates the **N** subjects/options forming the sample of measurement “ m ”—then, to compare the empirical probability from “ N_{mi} ” ($p(x_i/m_i)$) with any other empirical probability from “ N_m ” ($p(x/m)$), it is **necessary** to take into account the **effect of “ N_m ”** on the empirical probability from measurement “ N_{mi} ”. Since the effect of **N** is **inversely proportional** to the value of **N** , logically, if “ N_{mi} ” differs from “ N_m ”, the effect of **N** must equal the quotient **N_{mi} divided by N_m** .

16.4.1.12. The effect produced by “ N_m ” on the empirical probability “ $p(x_i/m_i)$ ” is inversely proportional to “ N_m ”; therefore the effect of **N** will equal dividing the **N** of measurement “ m_i ”—that is, dividing “ N_{mi} ”—by the **N** of measurement “ m ”, that is, dividing “ N_{mi} ” by “ N_m ”.

Effect of $N = N_{mi} : N_m$

16.4.1.13. The effect of **N** is **multiplied** by the factor belonging to measurement “mi” in order to compare it with any factor from measurement “m”.

Effect of N on $p(x_i/m_i) = p(x_i/m_i) \cdot (N_{mi} : N_m)$

16.4.1.14. If one wishes to know the **differential** of an empirical probability from measurement “mi” relative to an empirical probability from measurement “m”—with **N** of “mi” different from **N** of “m”—then it is necessary to **multiply the effect of N** by the empirical probability from “mi” that will be related to the empirical probability belonging to “m”:

$(p(x_i/m_i) \cdot (N_{mi} : N_m)) - p(x/m) = \text{differential, of increase or decrease, between subjects or options from measurements with different } \mathbf{N}.$

16.4.2. Study of the proposition “ $N_{mi} : N_m$ ”

16.4.2.1. Empirical explanation

16.4.2.1.1. Let’s take an example with **$N_{mi} = 10$** ; that is, in sample “mi” the magnitude **N** totals ten subjects or options, and suppose in this example the empirical probability **closest to the theoretical probability** is exactly equal to the inversion of “ N_{mi} ”; that is, the closest empirical probability of “mi”, **$p(x_i \approx /m_i)$** , is identical to the inversion of “ N_{mi} ”, i.e., **$1/N_{mi}$** . (This is just for ease of illustration; in general, the empirical probability closest to the inversion of **N** need not equal the inversion of **N**.)

N_{mi} = total number of subjects or options in the sample of measurement “mi”.

16.4.2.1.1.1. Suppose the sample “N_{mi}” equals ten subjects or options:

N_{mi} = 10 subjects or options form **N** in measurement “mi”.

16.4.2.1.1.2. The inversion of “N_{mi}” will be:

$$1/N_{mi} = 1 : 10 = 0.1$$

16.4.2.1.1.3. Suppose that the probability **closest to the inversion of N** in measurement “mi” has an empirical probability exactly equal to the inversion of **N** in “mi”; then its empirical probability equals the inversion of “N_{mi}”—i.e., the empirical probability closest to the inversion of “N_{mi}” happens, by coincidence in this example, to equal the theoretical probability of measurement “mi”.

$$p(x_i \approx /mi) = 0.1 = 1/N_{mi} = \text{closest probability in } N_{mi} \text{ to the inversion of } N \text{ in this example.}$$

16.4.2.1.1.4. The fact that the closest empirical probability of “N_{mi}” equals the inversion of “N_{mi}” implies that this empirical probability has **Bias Level zero** and **Similarity Level zero**.

16.4.2.1.1.5. Conclusion: the closest empirical probability in measurement “N_{mi}”, in this example, will have **Bias Level = 0** (minimum possible bias) and **Similarity Level = 0** (maximum possible similarity).

16.4.2.1.2. Consider another case: now an **arbitrary measurement “m”** where the magnitude of **N** is **one hundred**, and the empirical probability **closest to the theoretical probability** also, by coincidence in this example, equals the inversion of “N_m”.

16.4.2.1.2.1. We start with an arbitrary measurement “m” whose **N** equals one hundred subjects or options:

$$Nm = 100$$

16.4.2.1.2.2. The inversion of **N** for measurement “m” will be:

$$1/Nm = 1 : 100 = 0.01$$

16.4.2.1.2.3. Thus, again by coincidence, in this arbitrary measurement “m” there exists a subject/option whose empirical probability is exactly equal to the inversion of “Nm”; i.e., the empirical probability closest to the inversion of “Nm” equals **1/Nm**.

$$p(x \approx /m) = 0.01 = 1/Nm \text{ [in this example, just to explain the logic]}$$

16.4.2.1.2.4. The fact that the closest empirical probability in “Nm” equals the inversion of “Nm” implies that this empirical probability has **Bias Level zero** and **Similarity Level zero**.

16.4.2.1.3. Inter-individual, inter-measurement study of the probabilities closest to the theoretical probability in measurements “Nmi = 10” and “Nm = 100”

16.4.2.1.3.1. To relate the probability closest to the theoretical probability of measurement “mi”, with **N = 10**:

$$Nmi = 10$$

$$1/Nmi = 0.1$$

16.4.2.1.3.1.1. Where the closest probability to the theoretical probability in “mi” equals the theoretical probability of “mi”:

$$p(x_i \approx / m_i) = 0.1$$

16.4.2.1.3.1.2. Relating this closest empirical probability from “Nm_i” to the closest empirical probability to the theoretical probability in an arbitrary measurement “m” with **N = 100**:

$$N_m = 100$$

$$1/N_m = 0.01$$

16.4.2.1.3.1.3. Where the closest probability to the theoretical probability in that arbitrary measurement equals the inversion of its own “Nm”, with this “Nm” different from “Nm_i”:

$$p(x \approx / m) = 0.01$$

16.4.2.1.3.1.4. It will be absolutely necessary to take into account the **effect of “Nm”** on the empirical probability of “Nm_i” if one wishes to relate the closest-to-inversion probabilities from the two different measurements. This effect of “Nm” on “p(x_i/m_i)” is the **effect of N**—in this case, the effect of “Nm” on the closest empirical probability to the theoretical probability in measurement “Nm_i”.

16.4.2.1.3.2. If the way “Nm” relates to whatever it acts upon is **inverse proportionality**, one must study the **inverse proportional relationship** over that which it relates to.

16.4.2.1.3.3. If the empirical probability of “Nmi” was calculated on “Nm”, then the inverse proportional relationship of “Nm” over an empirical probability from “Nmi” will be **inverse to “Nm”**. If the aim is to know **what the empirical probability of “Nmi” would be were it in measurement “Nm”**, and the tendency of any empirical probability toward zero is inversely proportional to **N**, then the tendency toward zero of the empirical probability of “Nmi” **in “Nm”** should be inversely proportional to **Nm**. Therefore, to know **in what proportion** this tendency toward zero should occur, one must **multiply the effect of N** by the empirical probability of “Nmi”.

16.4.2.1.3.4. The inverse-proportional relationship of **“Nm”** on the empirical probability of **“Nmi”** will be an inverse-proportional relationship of **“Nm”** on the empirical probability of **“Nmi”** equal to the **product** of the empirical probability from measurement “mi” by the **quotient Nmi / Nm**—that is, the **effect produced by N** on the empirical probability of “Nmi”.

Effect of “Nm” on “p(xi/mi)” = $p(xi/mi) \cdot (Nmi : Nm)$

16.4.2.1.3.5. If one can compute the effect produced by **N** on an empirical probability when relating empirical probabilities from samples with **different N**, then one can compare empirical probabilities from measurements taken on samples with different **N**.

16.4.2.1.3.6. If we have the empirical probability **closest to the theoretical probability** in a measurement “Nmi” whose **N** is, for example, **ten** subjects or options, and we want to compare this closest empirical probability of “Nmi” against the closest empirical probability to the theoretical probability of some other measurement “Nm” where, say, **Nm = 100** subjects or options, then the comparison of empirical probabilities will be equal to comparing the **Effect of N** applied to the empirical probability of “Nmi”, and contrasting that product **differentially** with the empirical probability of measurement “Nm”. This comparison yields the **level of increase or decrease** in the **real variation** between both probabilities with different sample **N**. In this specific case, since both

empirical probabilities in each sample (with different **N**) were **identical to the inversion of N** (by coincidence), then—applying the Effect of N to the closest-to-inversion probability of measurement “mi”—the product is an empirical probability **identical** to the inversion of “Nm”; therefore, subtracting from it the “Nm” probability (also identical to the inversion of “Nm”) logically yields **zero**.

$$\{ [p(x_{i\approx}/m_i) = 0.1] \cdot [(N_{mi} = 10) / (N_m = 100)] = 0.01 \} - [p(x_{\approx}/m) = 0.01] = 0$$

16.4.2.1.3.6.1. Simplifying in the example (removing some brackets for clarity) and leaving the values below as a legend:

$$[0.1 \cdot (10 / 100)] - 0.01 = 0$$

$$p(x_{i\approx}/m_i) = 0.1$$

$$N_{mi} = 10$$

$$N_m = 100$$

$$p(x_{\approx}/m) = 0.01$$

16.4.2.1.3.6.2. Synthesis: To compute the **real variation** between two empirical probabilities from measurements with **different N**, divide the **N** of the **first** measurement by the **N** of the **second** measurement and **multiply the quotient by the first measurement’s empirical probability**; or (symmetrically) multiply the **second** measurement’s probability by the quotient of its **N** over the first measurement’s **N**, depending on which simplifies later calculations (as explained below), in order to

compare both empirical probabilities. In the next example—purely illustrative—the Effect of N will be applied to the **first** measurement.

$p(x_i/2^{nd}-m)$ = empirical probability in the **second** measurement, labeled “m”

$p(x_i/1^{st}-m_i)$ = empirical probability in the **first** measurement, labeled “mi”

N_{mi} = value of **N** in the **first** measurement “mi”

N_m = value of **N** in the **second** measurement “m”

16.4.2.1.3.6.2.1. Real increase between two measurements

$p(x_i/2^{nd}-m) - [p(x_i/1^{st}-m_i) \cdot (N_{mi} : N_m)] = \text{positive} \rightarrow \text{growth}$

16.2.2.1.3.6.2.2. Real decrease between two measurements

$[p(x_i/1^{st}-m_i) \cdot (N_{mi} : N_m)] - p(x_i/2^{nd}-m) = \text{positive} \rightarrow \text{decrease}$

16.4.2.1.3.7. Whenever, in two measurements with different **N**, the **closest** probabilities to their respective inversions of **N** happen (by coincidence) to be **equal to the inversion of N** in their respective samples, then applying the Effect of N to one of those closest-to-inversion probabilities will yield a **zero differential**. Logically—regardless of **N**—if two empirical probabilities equal their inversions of **N**, then after applying the Effect of N to one of them (each identical to the inversion of **N** in its own sample), the adjusted probability becomes **equal to the inversion of N** of the other sample. Hence, its difference from that other sample’s probability (which is identical to

its own inversion of **N**) is **zero**. Intuitively: independent of **N**, if two empirical probabilities (from different samples **N**) are each identical to their **theoretical** probabilities, there is no reason to assign a greater empirical value to one than to the other once the Effect of N is applied, because both reflect **equality of opportunities** (chance) and **not bias**. Therefore, with different **N** samples, when each empirical probability equals chance behaviour, the **differential must be zero** after applying the Effect of N to one of them.

16.4.2.1.3.8. Whenever empirical probabilities from measurements with **different sample N** are compared, **one of the two empirical probabilities must be multiplied** by the quotient of **its sample N divided by the other sample's N**.

16.4.2.1.3.9. Conclusion: It is **only** possible to compare probabilities from measurements with **different N** if **one** of the two probabilities is multiplied by the result of **dividing its sample's N by the other (different) sample's N**; only **then** can the two probabilities be related, even if they come from measurements with different **N**.

16.4.2.2. Theoretical explanation of the Effect of N in this example

16.4.2.2.1. In the example, the closest empirical probability to the theoretical probability in a measurement “Nmi” with **ten** subjects/options equals the inversion of

“Nmi”=10

$p(x_i \approx m_i) = 1 / (N_{mi} = 10) = 0.1$ [in this example]

16.4.2.2.1.1. And we also have, in some arbitrary measurement “Nm” with **100** subjects/options, the closest empirical probability to the theoretical probability **equal** to the inversion of “Nm”:

$p(x \approx m) = 1 / (N_m = 100) = 0.01$

16.4.2.2.2. If **N** increases, one expects that, as **N** increases, the **similarity** of the whole sample increases and the **bias** decreases.

16.4.2.2.3. If **N** increases, then one projects that the **theoretical probability** of “**N_{mi}**”, upon increasing “**N_{mi}**”, increases similarity and reduces bias; and if, with “**N_{mi} = 10**”, the closest-to-theoretical probability is in fact **equal** to the theoretical probability in the example, one expects this **equality** to be **maintained** if the value of “**N_{mi}**” is raised.

16.4.2.2.4. If the value of “**N_{mi}**” were raised to a value “**N_m**”, then normally the inversion of “**N_{mi}**” would tend to equal the inversion of “**N_m**”. If “**N_m = 100**”, the inversion is **0.01**; thus, raising “**N_{mi}**” from **10** to **100** would make the inversion of “**N_{mi}**” equal to the inversion for **100**.

16.4.2.2.5. If the closest-to-inversion probability in “**N_{mi} = 10**” equals **1/N_{mi}**, one expects that, as “**N_{mi}**” increases, the closest probability to inversion in “**N_{mi}**” will **continue to equal 1/N_{mi}**.

16.4.2.2.6. If “**N_{mi} = 10**” and “**N_m = 100**”, then one expects that the closest probability of “**N_{mi}**”—equal to **1/10**—would, if **N_{mi}** were **100**, become the **1/100** inversion.

16.4.2.2.7. Synthesis: If increasing **N** is expected to **reduce empirical probabilities**, then whenever **N_m > N_{mi}**, the quotient **N_{mi} / N_m** gives the **proportion of reduction** of empirical probabilities. Hence, **multiplying each empirical probability by N_{mi} / N_m** yields the empirical probability that the subject/option **would** have if the sample were increased.

16.4.2.2.8. Therefore, when **N_m > N_{mi}**, the quotient **N_{mi} : N_m** tends to measure the **Effect of N** on the empirical probabilities of measurement “**mi**”, as the **proportion of reduction** in the magnitude of the empirical probability as **N** increases.

Effect of **N** on “**mi**” = **N_{mi} : N_m**

16.4.2.2.9. Thus, **any empirical probability** from “mi” multiplied by the quotient of its **N** in “mi” over a **larger N** from some other measurement “m” equals the **proportion by which** the empirical probability will tend to **decrease** due to the Effect of N.

16.4.2.3. The Effect of N is the effect of the inversion of the change in sample magnitude on empirical probabilities

16.4.2.3.1. Likewise, suppose that **N_m** is **less** than **N_{mi}**. Just as empirical probabilities are expected to **decrease** when **N** increases, they are expected to **increase** when **N** decreases; therefore the quotient **N_{mi} / N_m**, when **N_m < N_{mi}**, would be the **proportional increase** in empirical probabilities due to the reduction in **N**.

16.4.2.3.2. Thus, if **N_m < N_{mi}**, one expects the probabilities in measurement **N_m** to be **greater** than in **N_{mi}**:

$$N_m < N_{mi} \rightarrow p(x+/m) > p(x_i+/mi)$$

$$N_m < N_{mi} \rightarrow p(x+/-/m) > p(x_i+/-/mi)$$

$$N_m < N_{mi} \rightarrow p(x\approx/m) > p(x_i\approx/mi)$$

16.4.2.3.3. Thus, the Effect of **N** on “mi”, when in a measurement “N_m” the magnitude “N_m” is **less** than in measurement “N_{mi}”, will **still** be equal to the quotient **N_{mi} / N_m**.

$$\text{Effect of N on “mi”} = N_{mi} : N_m$$

16.4.2.3.4. Therefore, the way to calculate the effect of the magnitude **N** on the **empirical probabilities** of a measurement “mi”, when **N decreases**, is likewise by the **product** of the empirical probabilities of “mi” by the Effect of N, with the Effect of N equal to the quotient of the magnitude **N** in measurement “mi” over the magnitude **N** of measurement “m”.

Effect of N on the empirical probability of “mi” = $p(x_i/m_i) \cdot (N_{mi} : N_m)$

16.4.3. Conclusion on the Effect of N

16.4.3.1. Given two measurements from samples with **different N**, where one measurement “mi” has a magnitude **N** different from measurement “m” (whether larger or smaller), the Effect of **N** on the empirical probabilities of measurement “mi” will **always** be the same: the **product** of the empirical probability of the designated measurement “mi” by the **quotient** of the magnitude **N** of “mi” over the variable magnitude **N** of the designated measurement “m”.

16.4.3.1.1. The **Effect of N** is thus understood, on whatever you want to know its effect, as the **product** of (that quantity) by the **quotient** of **N** in measurement “mi” over the variable **N** of the other (different-**N**) measurement “m”. “mi” is the measurement to which the factor under study (for the Effect of N) belongs.

Effect of N = $N_{mi} : N_m$

16.4.3.1.2. Effect of N on the empirical probability of “mi” = product of the empirical probability of measurement “mi” by the quotient N_{mi} / N_m of any other measurement “m”.

Effect of N on the empirical probability of “mi” = $p(x_i/m_i) \cdot (N_{mi} : N_m)$

16.4.3.1.3. Once the Effect of N is established, if there are **M** measurements, one can compute **inter-measurement descriptive statistics**. To do so, among the **M** measurements designate one particular measurement as the reference “m” (e.g., let the **first** measurement be “m”), to estimate **real variations** of the other m-th measurements “me” [in Spanish, I refer “me” as *mediciones emesimas*= *m-th*

measurements]”, for the **mean** and **mean differential** of real variations **between** the first (reference) measurement “m” and **all** the other measurements, each designated “mi”.

16.4.3.1.3.1. Real Individual Variation relative to “p(x/m)” =

$$[p(x_i/m_i) \cdot (N_{mi} : N_m)] - p(x/m)$$

16.4.3.1.3.2. Signed Mean Real Variation relative to “p(x/m)”.

Because we average the differentials (taking into account the Effect of N) between the empirical value of the **reference** measurement “Nm” and **all** empirical values of the other measurements “Nmi”, the summation must be averaged over **M – 1 (M= total number of measurements]**, since one differential (the reference measurement with itself) is excluded. Thus, the sum of inter-measurement differentials, considering the Effect of N, is averaged over **M – 1**.

16.4.3.1.3.2.1. Signed Mean Real Variation — growth =

$$\sum \{ [p(x_i/m_i - m_i) \cdot (N_{mi} : N_m)] - p(x/1^{st}-m) \} : (M - 1) = \text{positive} \rightarrow \text{accepted}$$

where:

$p(x_i/m_i - m_i)$ = empirical probability of each m-th measurement (each designated “mi”)

$p(x/1^{st}-m)$ = empirical probability of the **first** (reference) measurement “m”

M = total number of measurements

Nm = first (reference) measurement

Nmi = the **N** of the m-th measurement to which $p(x_i/m_i - m_i)$ belongs

16.4.3.1.3.2.2. Signed Mean Real Variation — decrease =

$$\sum \{ p(x/1st-m) - [p(xi/me-mi) \cdot (Nmi : Nm)] \} : (M - 1) = \text{positive} \rightarrow \text{accepted}$$

16.4.3.1.3.3. Absolute Mean Real Variation =

$$\sum | \{ [p(xi/me-mi) \cdot (Nmi : Nm)] - p(x/m) \} | : (M - 1)$$

16.4.3.1.3.4. Mean Deviation of the Real Variation [meta deviation] =

$$\sum / \{ | \{ [p(xi/me-mi) \cdot (Nmi : Nm)] - p(x/m) \} | -$$

$$\{ \sum | \{ [p(xi/me-mi) \cdot (Nmi : Nm)] - p(x/m) \} | : M-1 \} / : (M - 1)$$

16.5. Contrasts of Possible Variation in inter-measurement studies with different N, applying the Effect of N

16.5.1. Concept

16.5.1.1. Inter-measurement studies may be **between different measurements with the same sample N**, or **between different measurements with different sample N**.

16.5.1.2. When the inter-measurement studies are **with the same N**, empirical probabilities from each measurement (with the same N) may be related to any other empirical probability from any other same-**N** measurement.

16.5.1.3. When the inter-measurement studies are **with different N**, one must **multiply the Effect of N** by the empirical probabilities of the measurement designated “mi” whose relationship to another measurement “m” (with different N) is sought.

16.5.1.4. In **intra-individual inter-measurement** studies—where we want the **differential relation** of the same subject/option across two different measurements—

if those measurements also differ in **N**, then whichever is designated “mi” (first or second—scientific policy decides), its empirical probability must be multiplied by the **Effect of N**. Thus, the inter-measurement intra-individual Effect of N equals the **Effect of N times the empirical probability** of the measurement designated “mi” (whether that is the first or the second measurement, as decided).

16.5.1.5. To **simplify equations**, we will apply the Effect of N to the **second** measurement’s empirical probabilities:

Effect of N on second measurement = $p(x_i/2^{nd-mi}) \cdot (N_{mi} : N_m)$

16.5.1.6. In any case, if the Effect of N is applied to a measurement (first or second), it must be understood that the Effect of N should be applied **not only to empirical probabilities**. If it is necessary to compare **empirical dispersion** of samples with **different N**, one must compute the **Effect of N on the Mean or Standard Deviation (DMoS)**, which will be required in the **Sample Possible Variation** sections:
Effect of N on Mean or Standard Deviation (DMoS), if necessary =

$DMoS_{-mi} \cdot (N_{mi} : N_m)$

$DMoS_{-mi}$ = Mean or Standard Deviation of sample “mi”

16.5.1.7. The reason to apply the Effect of N to the **Mean or Standard Deviation (DMoS)**, if needed, is that **dispersion** (DM or S) is a variable dependent—not only on the object of study and the distribution of raw scores/frequencies—but also on **N**. If **N increases**, dispersion tends to **decrease**; if **N decreases**, dispersion tends to **increase**. This also holds in **limited-option** models, especially when **N** is variable, or when the empirical hypothesis’ **N** varies materially or socially [or technologically]. Since DM or S **depends on N** (among other factors), if we wish to contrast, for instance,

empirical dispersion across measurements with different N, it is necessary to apply the **Effect of N** to the empirical dispersion of those measurements designated “mi”.

16.5.2. Contrasts of Individual Possible Variation in samples with different N

16.5.2.1. Contrasts of Individual Critical Possible Variation in samples with different N

16.5.2.1.1.

Legend

$p(x_i/1^{st}-m)$ = empirical probability of the subject/option in the **first** measurement, designated “m”

$p(x_i/2^{nd}-m_i)$ = empirical probability of the subject/option in the **second** measurement, designated “mi”

16.5.2.1.1.1. Contrast of Individual Critical Possible Variation by Chance in equality of opportunities,
if the **absolute bias** of the **second** measurement **after applying the Effect of N** is **less than** the absolute bias of the **first** measurement:

$$[| p(x_i/1^{st}-m) - 1/Nm | \cdot p(x_c)] - \{ | p(x_i/1^{st}-m) - 1/Nm | - [p(x_i/2^{nd}-m_i) \cdot (N_{mi} : N_m)] - 1/Nm | \}$$

= **zero or positive** → **increase in equality accepted by chance**

$$p(x_c) = X : 100$$

X = error percentage

16.5.2.1.1.2. Contrast of Individual Critical Possible Variation by Chance in the study of positive bias, only if the product (Effect of N × second-measurement empirical probability) is **greater** than the first-measurement empirical probability:

$[(1 - p(xi/1st-m)) \cdot p(xc)] - \{ [p(xi/2nd-mi) \cdot (Nmi : Nm)] - p(xi/1st-m) \} = \text{zero or positive} \rightarrow \text{increase of positive bias accepted by chance}$

$p(xc) = X : 100$

X = error percentage

16.5.2.1.1.3. Contrast of Individual Critical Possible Variation by Chance, in the study of negative bias, only if the empirical probability of the first measurement is greater than the **product** of the second measurement's empirical probability by the Effect of N:

$((p(xi/1st-m) - 0) \cdot p(xc)) - \{ p(xi/1st-m) - [p(xi/2nd-mi) \cdot (Nmi : Nm)] \} =$
 zero or positive $\rightarrow$ increase of negative bias accepted by chance
 $p(xc) = X : 100$
 X = error percentage

16.5.2.1.2. Individual Critical Reliable Possible Variation

16.5.2.1.2.1. Contrast of Individual Critical Reliable Possible Variation, in the study of equality of opportunities:

$$\{ | p(x_i/1st-m) - 1/N_m | - [p(x_i/2nd-m_i) \cdot (N_{mi} : N_m)] - 1/N_m | \} - \{ | p(x_i/1st-m) - 1/N_m | \cdot p(x_c) \}$$

= **zero or positive** → **reliable increase of equality accepted**

$$p(x_c) = \frac{X}{100}$$

X = reliability percentage

16.5.2.1.2.2. Contrast of Individual Critical Reliable Possible Variation, in the study of positive bias:

$$\{ [p(x_i/2nd-m_i) \cdot (N_{mi} : N_m)] - p(x_i/1st-m) \} - \{ (1 - p(x_i/1st-m)) \cdot p(x_c) \} = \text{zero or positive} \rightarrow \text{reliable increase of positive bias accepted}$$

$$p(x_c) = \frac{X}{100}$$

X = reliability percentage

16.5.2.1.2.3. Contrast of Individual Critical Reliable Possible Variation, in the study of negative bias:

$\{ p(xi/1st-m) - [p(xi/2nd-mi) \cdot (Nmi : Nm)] \} - \{ (p(xi/1st-m) - 0) \cdot p(xc) \} = \text{zero or positive} \rightarrow \text{reliable increase of negative bias accepted}$

$p(xc) = X : 100$

X = reliability percentage

16.5.2.2. Empirical Possible Variation in samples with different N

16.5.2.2.1. Individual Empirical Possible Variation by Chance

16.5.2.2.1.1. Individual Empirical Possible Variation by Chance via the **First Modality of Empirical Error**, in equality of opportunities, and **only** if the **second** measurement (after applying the Effect of N) has an **absolute bias** less than that of the **first** measurement:

$(|p(xi/1st-m) - 1/Nm| \cdot \alpha) - \left\{ |p(xi/1st-m) - 1/Nm| - \left| [p(xi/2nd-mi) \cdot (Nmi : Nm)] - 1/Nm \right| \right\} = \text{zero or positive} \rightarrow \text{accepted as increased equality by chance}$

16.5.2.2.1.2. Individual Empirical Possible Variation by Chance via the **Second Modality of Empirical Error**, **only and exclusively** if the object of study is to **tend to the maximum possible dispersion**.

16.5.2.2.1.2.1. Contrast (positive bias), via the Second Modality of Empirical Error, **only if** the Effect of N times the second measurement's probability is **greater** than the first measurement's probability:

$$\left((1 - p(xi/1st-m)) \cdot \neg\beta \right) - \left\{ [p(xi/2nd-mi) \cdot (Nmi : Nm)] - p(xi/1st-m) \right\} = \text{zero or positive} \rightarrow \text{increase of positive bias accepted by chance}$$

16.5.2.2.1.2.2. Contrast (negative bias), via the Second Modality of Empirical Error, **only if** the first measurement's empirical probability is **greater** than the **product** (Effect of N × second measurement's probability):

$$\left((p(xi/1st-m) - 0) \cdot \neg\beta \right) - \left\{ p(xi/1st-m) - [p(xi/2nd-mi) \cdot (Nmi : Nm)] \right\} = \text{zero or positive} \rightarrow \text{increase of negative bias accepted by chance}$$

16.5.2.2.2. Individual Empirical Reliable Possible Variation

16.5.2.2.2.1. Via the **First Modality of Empirical Reliability**, in equality of opportunities:

$$\{ | p(xi/1st-m) - 1/Nm | - | [p(xi/2nd-mi) \cdot (Nmi : Nm)] - 1/Nm | \} - \{ | p(xi/1st-m) - 1/Nm | \cdot \neg\alpha \}$$

= zero or positive → reliable increase of equality accepted

16.5.2.2.2.2. Via the **Second Modality of Empirical Reliability**, if the object of study is **maximum possible dispersion**.

16.5.2.2.2.2.1. Contrast (positive bias), via the Second Modality of Empirical Reliability:

$\{ [p(\text{xi}/2\text{nd}-\text{mi}) \cdot (N_{\text{mi}} : N_{\text{m}})] - p(\text{xi}/1\text{st}-\text{m}) \} - \{ (1 - p(\text{xi}/1\text{st}-\text{m})) \cdot \beta \} = \text{zero or positive}$
→ reliable increase of positive bias accepted

16.5.2.2.2.2.2. Contrast (negative bias), via the Second Modality of Empirical Reliability:

$\{ p(\text{xi}/1\text{st}-\text{m}) - [p(\text{xi}/2\text{nd}-\text{mi}) \cdot (N_{\text{mi}} : N_{\text{m}})] \} - \{ (p(\text{xi}/1\text{st}-\text{m}) - 0) \cdot \beta \} = \text{zero or positive}$
→ reliable increase of negative bias accepted

16.6. Possible Variation — by Chance or Reliable, Critical or Empirical — at the Sample level

16.6.1. Just as one can compute **Individual** Possible Variation (by Chance or Reliable; Critical or Empirical), one can likewise compute **Sample** Possible Variation (by Chance or Reliable; Critical or Empirical). The difference is that, at the **sample** level, what is compared is the **sample tendency** directly (rather than individual tendencies), the sample tendency being, in essence, the **empirical dispersion** of the sample.

16.6.2. The **necessary variation**, in sample terms, is the variation required to **increase or decrease the sample variation**. Ideally, in the **bias** study, one increases variation toward **Maximum Theoretical Possible Mean Deviation**; and, in **equality of opportunities**, the ideal is a general tendency of the sample dispersion toward **zero**. To simplify examples—and given earlier explanations of the predominance that **Mean Deviation** has over **Standard Deviation** in *Impossible Probability*—the following examples will be developed using **Mean Deviation (DM)**. It is always understood that the Second Method can be adapted to the First Method when necessary: one simply substitutes **DM** with **S**, and, where required, substitutes **Maximum Theoretical**

Possible Mean Deviation with Maximum Theoretical Possible Standard Deviation.

Another reason to proceed only with **DM** is brevity.

16.6.3. Given a **normal Mean Deviation** of a sample, in a **bias study**, the **necessary variation** to tend the normal Mean Deviation to the **Maximum Theoretical Possible Mean Deviation** equals:

$$\{ [(1 - 1/N) \cdot 2] : N \} - \text{DM} = \text{Necessary variation to Maximum Theoretical Possible Mean Deviation (from the normal Mean Deviation)}$$

DM = Mean Deviation

16.6.4. Given a **normal Mean Deviation** of a sample, in **equality of opportunities**, the **necessary variation** to tend the normal Mean Deviation to **zero dispersion** equals the difference of the normal Mean Deviation minus zero; that is, the **Mean Deviation itself** is the necessary variation by which the sample's empirical dispersion must decrease to reach zero:

$$\text{Necessary variation to zero of the normal Mean Deviation} = \text{DM} - 0 = \text{DM}$$

DM = Mean Deviation

16.6.5. Once the **necessary variations** are computed, one can compute the **Possible Variation** (by Chance or Reliable; Critical or Empirical), whether in the study of **bias** or **equality of opportunities**.

16.6.6. Sample-level Possible Variation—by Chance or Reliable; Critical or Empirical—based on the necessary variation to Maximum Theoretical Possible Mean Deviation, in the bias study.

16.6.6.1. Sample Critical Possible Variation by Chance on the necessary variation in a bias study:

$$\{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot p(xc)$$

$$p(xc) = X : 100$$

X = error percentage

16.6.6.2. Normal Sample Empirical Possible Variation by Chance on the necessary variation when the object of study is **maximum dispersion** (Second Modality of Empirical Error):

$$\{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot \neg\beta$$

16.6.6.3. Normal Sample Critical Reliable Possible Variation on the necessary variation in a bias study:

$$\{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot p(xc)$$

$$p(xc) = X : 100$$

X = reliability percentage

16.6.6.4. Sample Empirical Reliable Possible Variation on the necessary variation when the object of study is **maximum dispersion** (Second Modality of Empirical Reliability):

$$\{ \{ [(1 - 1/N) \cdot 2] : N \} - DM \} \cdot \beta$$

16.6.7. Sample Possible Variation—by Chance or Reliable; Critical or Empirical—normal in the study of equality of opportunities
Necessary variation in DM for equality of opportunities = $DM - 0 = DM$

16.6.7.1. Normal Sample Critical Possible Variation by Chance in equality of opportunities:

$$(DM - 0) \cdot p(xc) = DM \cdot p(xc)$$

$$p(xc) = (X : 100)$$

X = error percentage

16.6.7.2. Normal Sample Empirical Possible Variation by Chance in equality of opportunities (First Modality of Empirical Error):

$$DM \cdot \alpha$$

16.6.7.3. Sample Critical Reliable Possible Variation in equality of opportunities:

$$DM \cdot p(x_c)$$

$$p(x_c) = \frac{X}{100}$$

X = reliability percentage

16.6.7.4. Normal Sample Empirical Reliable Possible Variation in equality of opportunities (First Modality of Empirical Reliability):

$$DM \cdot \neg\alpha$$

16.6.8. Contrasts of Sample Possible Variation—by Chance or Reliable; Critical or Empirical

16.6.8.1. Inter-measurement contrasts will differ depending on whether the different measurements to be contrasted **do or do not** have the same variable magnitude **N**.

16.6.8.2. If the different measurements to be contrasted have the **same N**, then the **direct differential** of the samples' empirical dispersions is compared **directly** against the **Possible Variation**.

16.6.8.3. If the different measurements to be contrasted **differ in N**, then one of the two measurements must be designated as “**mi**”; the most rational choice is to designate as “**mi**” the measurement that **simplifies the equations** when so designated.

16.6.8.4. In the **Individual** Possible Variation critique (by Chance; Critical or Empirical) it was shown that, in these contrasts, the most rational choice is to designate the **second** measurement as “**mi**”; therefore the second measurement will be “**mi**”, and the first measurement will be designated “**m**”.

16.6.8.5. In **different-N** inter-measurement contrasts, **all statistics** (individual or sample) belonging to the measurement designated “**mi**” must be **multiplied by the Effect of N** in order to be compared with the (individual or sample) statistics of the measurement designated “**m**”.

16.6.8.6. In what follows, we will first present the **Possible Variation contrasts of sample dispersion** for inter-measurement studies **with the same N**; afterwards, we will present the **same contrasts** but **including the Effect of N** on the second measurement (designated “**mi**”) for inter-measurement studies on measurements with **different sample N**.

16.6.8.7. In Possible Variation contrasts by Chance—Critical or Empirical; Bias or Equality—it is **absolutely necessary beforehand** that: either **in a bias study** the **second** measurement’s sample dispersion is **greater** than the first’s, **or in equality of opportunities** the **first** measurement’s sample dispersion is **greater** than the second’s. **Only if this prerequisite is met** may the Sample Possible Variation contrasts (by Chance; Critical or Empirical) be applied; otherwise they are **not applied**. The purpose of this contrast is: once a **real variation** has been observed **increasing bias** (second-measurement dispersion greater than first), **or** once a real variation **increasing equality** has been observed (second-measurement dispersion **less** than first), then—and **only** then—apply, for each object of study, the **Sample Possible Variation** contrast (by Chance; Critical or Empirical).

16.7. Contrasts of Sample Possible Variation—by Chance or Reliable; Critical or Empirical—normal, in inter-measurement studies with equal N

16.7.1. Normal Sample Critical Possible Variation by Chance on the necessary variation, in **bias** and in **equality of opportunities**, between measurements with equal **N**.

16.7.1.1. Normal Sample Critical Possible Variation by Chance in a **bias** study, between measurements with equal **N**, **only if** the **second** measurement's dispersion is **greater** than that observed in the **first**:

$$\{ \{ [(1 - 1/N) \cdot 2] : N \} - DM/1^a \} \cdot p(xc) \} - (DM/2^a - DM/1^a) = \text{zero or positive} \rightarrow \text{increase}$$

of bias accepted by chance

$$p(xc) = X : 100$$

$$X = \text{error percentage}$$

$$DM/2^a = \text{Mean Deviation of the second measurement}$$

$$DM/1^a = \text{Mean Deviation of the first measurement}$$

16.7.1.2. Normal Sample Critical Possible Variation by Chance in **equality of opportunities**, between measurements with equal **N**, **only if** the dispersion observed in the **second** measurement is **less** than that observed in the **first**:

$(DM/1^a \cdot p(xc)) - (DM/1^a - DM/2^a) = \text{zero or positive} \rightarrow \text{increase in equality accepted by chance}$

$p(xc) = X : 100$

X = error percentage

16.7.2. Normal Sample Empirical Possible Variation by Chance in **bias or equality of opportunities**, between measurements with equal **N**

16.7.2.1. Sample Empirical Possible Variation by Chance in a **bias** study, between measurements with equal **N** (Second Modality of Empirical Error), and **only if** the second measurement's sample dispersion is **greater** than the first's:

$\{ \{ [(1 - 1/N) \cdot 2] : N \} - DM/1^a \} \cdot \neg \beta - (DM/2^a - DM/1^a) = \text{zero or positive} \rightarrow \text{increase of bias accepted by chance}$

16.7.2.2. Sample Empirical Possible Variation by Chance in **equality of opportunities**, between measurements with equal **N** (First Modality of Empirical Error), and **only if** the second measurement's sample dispersion is **less** than the first's:

$(DM/1^a \cdot \alpha) - (DM/1^a - DM/2^a) = \text{zero or positive} \rightarrow \text{increase of equality accepted by chance}$

16.7.3. Sample Critical Reliable Possible Variation in bias and in equality of opportunities, between measurements with equal N

16.7.3.1. Normal Sample Critical Reliable Possible Variation in a bias study:

$(DM/2^a - DM/1^a) - \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM/1^a \} \cdot p(xc) = \text{zero or positive} \rightarrow \text{reliable}$
in bias accepted

$p(xc) = X : 100$

X = reliability percentage

16.7.3.2. Normal Sample Critical Reliable Possible Variation contrasts, in the study of equality of opportunities, between measurements with equal N

$(DM/1^a - DM/2^a) - [DM/1^a \cdot p(xc)] = \text{zero or positive} \rightarrow \text{reliable tendency toward}$
equality of opportunities accepted

$p(xc) = X : 100$

X = reliability percentage

16.7.4. Sample Empirical Reliable Possible Variation contrasts on the necessary variation, in the study of bias and in equality of opportunities, between measurements with equal N

16.7.4.1. Sample Empirical Reliable Possible Variation contrasts in equality of opportunities (First Modality of Empirical Reliability):

$(DM/1^a - DM/2^a) - \{DM/1^a \cdot \neg \alpha\} = \text{zero or positive} \rightarrow \text{reliable tendency toward equality of opportunities accepted}$

16.7.4.2. Normal Sample Empirical Reliable Possible Variation contrasts in the study of bias (Second Modality of Empirical Reliability):

$(DM/2^a - DM/1^a) - \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM/1^a \} \cdot \beta = \text{zero or positive} \rightarrow \text{accepted in bias study}$

16.8. Contrasts of Sample Possible Variation—by Chance or Reliable; Critical or Empirical—normal, in inter-measurement studies with different N

16.8.1. Normal Sample Critical Possible Variation by Chance contrasts, in a bias study, with different-N measurements, **only if** the sample dispersion in the **second** measurement, **after applying the Effect of N**, is **greater** than that observed in the **first** measurement:

$$\{ \{ [(1 - 1/N_m) \cdot 2] : N \} - DM/1^{a-m} \} \cdot p(x_c) - \{ [DM/2^{a-m_i} \cdot (N_{mi} : N_m)] - DM/1^{a-m} \}$$

= **zero or positive** → **increase of bias accepted by chance**

$$p(x_c) = X : 100$$

$$X = \text{error percentage}$$

$DM/2^{a-m_i}$ = Mean Deviation of the second measurement designated “mi”

$DM/1^{a-m}$ = Mean Deviation of the first measurement designated “m”

16.8.2. Normal Sample Empirical Possible Variation by Chance contrasts in a bias study, with different-N measurements (Second Modality of Empirical Error), **only if** the second measurement’s sample dispersion, after the Effect of N, is **greater** than that of the first:

$$\{ \{ [(1 - 1/N_m) \cdot 2] : N \} - DM/1^{a-m} \} \cdot \neg\beta - \{ [DM/2^{a-m_i} \cdot (N_{mi} : N_m)] - DM/1^{a-m} \} =$$

zero or positive → **increase of bias accepted by chance**

16.8.3. Normal Sample Critical Possible Variation by Chance contrasts in **equality of opportunities**, with different-N measurements, **only if** the second measurement's sample dispersion, after the Effect of N, is **less** than that observed in the first:

$$(DM/1^{a-m} \cdot p(xc)) - \{ DM/1^{a-m} - [DM/2^{a-mi} \cdot (N_{mi} : N_m)] \} = \text{zero or positive} \rightarrow$$

increase of equality accepted by chance

$$p(xc) = X : 100$$

X = error percentage

16.8.4. Normal Sample Empirical Possible Variation by Chance contrasts in **equality of opportunities**, with different-N measurements (First Modality of Empirical Error), **only if** the second measurement's sample dispersion, after the Effect of N, is **less** than that of the first:

$$(DM/1^{a-m} \cdot \acute{\alpha}) - \{ DM/1^{a-m} - [DM/2^{a-mi} \cdot (N_{mi} : N_m)] \} = \text{zero or positive} \rightarrow \text{increase of equality accepted by chance}$$

16.8.5. Normal Sample Critical Reliable Possible Variation contrasts, in the study of **bias** and in **equality of opportunities**, with different-N measurements

16.8.5.1. Sample Critical Reliable Possible Variation contrasts in a **bias** study:

$\{ [DM/2^{a-mi} \cdot (N_{mi} : N_m)] - DM/1^{a-m} \} - \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM/1^{a-m} \} \cdot p(xc) \} =$
zero or positive → accepted in bias study

$p(xc) = X : 100$

X = reliability percentage

16.8.5.2. Normal Sample Critical Reliable Possible Variation contrasts in **equality of opportunities**:

$\{ DM/1^{a-m} - [DM/2^{a-mi} \cdot (N_{mi} : N_m)] \} - [DM/1^{a-m} \cdot p(xc)] =$ **zero or positive → reliable tendency toward equality of opportunities accepted**

$p(xc) = X : 100$

X = reliability percentage

16.8.6. Normal Sample Empirical Reliable Possible Variation contrasts, in the study of **bias** or **equality of opportunities**, with different-N measurements

16.8.6.1. Sample Empirical Reliable Possible Variation contrasts in **equality of opportunities** (First Modality of Empirical Reliability):

$\{ DM/1^{a-m} - [DM/2^{a-mi} \cdot (N_{mi} : N_m)] \} - \{ DM/1^{a-m} \cdot \neg \acute{\alpha} \} = \text{zero or positive} \rightarrow \text{reliable}$
in equality of opportunities accepted

16.8.6.2. Normal Sample Empirical Reliable Possible Variation contrasts in a **bias** study (Second Modality of Empirical Reliability):

$\{ [DM/2^{a-mi} \cdot (N_{mi} : N_m)] - DM/1^{a-m} \} - \{ \{ [(1 - 1/N) \cdot 2] : N \} - DM/1^{a-m} \} \cdot \beta \} = \text{zero}$
or positive $\rightarrow$ reliable in bias study accepted

16.9. Interpretation of the Contrast of Sample Possible Variation—by Chance or Reliable; Critical or Empirical

16.9.1. Any **real variation** between two sample empirical dispersions from two different measurements will be accepted as **empirical by chance** if it is **equal to or less than** the **Sample Empirical Possible Variation by Chance**. This does **not** require it to be equal to or less than the **Sample Critical Possible Variation by Chance**, and **vice versa**. Thus, a change in behaviour in sample empirical dispersion may be **empirically by chance** but not **critical**, or **critical** but not **empirical**. Nevertheless, to accept that a change is **by chance both theoretically and empirically at the same time**, the **real variation** of the sample dispersion should be **equal to or less than** the **Sample Possible Variation by Chance, both Critical and Empirical**.

16.9.2. Any **real variation** between the dispersions of two different measurements—whether increasing **bias** or **equality of opportunities**—will be accepted as **empirically reliable** if it is **equal to or greater than** the **Sample Empirical Reliable Possible**

Variation. This does **not** require it to be equal to or greater than the **Sample Critical Reliable Possible Variation**, and **vice versa**. Thus, a change in the magnitude of sample dispersion may be **empirically** or **critically** reliable toward increasing bias or equality of opportunities; ideally, it would be **reliable both critically and empirically** to accept the tendency (bias or equality) in the sample.

16.9.3. The fact that a behavioural change is **not by chance** does **not** imply it is **reliable**. There exist behaviours that, while **not by chance**, are **not sufficiently reliable** either. It will be deemed **sufficiently reliable** only when the behavioural change is **equal to or greater than** the **Critical or Empirical Reliable Possible Variation**—ideally **both** critical and empirical. Conversely, the fact that a behavioural change is **not reliable** does not imply it is necessarily **random**: there are variables of **random natural origin** or **experimental (scientific/technological) origin** that transform into variables which, while **not sufficiently reliable**, also **do not** follow a purely **random** behaviour.